Soil Plasticity and the Structured Cam Clay Model



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Summary

I: Soil plasticity

II: The Structured Cam Clay model

III: Discussion on further development within the theoretical framework of the SCC model

Brief summary of the SCC model

The behaviour of soil found in nature differs remarkably from that of the same soil in laboratory reconstituted states.

When solving practical geotechnical engineering problems, the constitutive models developed for describing the behaviour of laboratory reconstituted soil are not good enough.

Influence of soil structure must be considered

- A simple predictive model
- the Structured Cam Clay (SCC) model

Aim of the model: to provide tool for the solution of boundary value problems encountered in geotechnical engineering.

Simple and convenient for engineers

We select Modified Cam Clay (MCC) model as the base for the new model.

(1) simple and rational, yet describes the behaviour reconstituted soil with acceptable accuracy;

(2) widely applied in geo-engineering field.

the SCC model, soil response defined in a 4-D space: *e*, current voids ratio, (*p'*, *q*), current stress state, stress history, and soil structure.

Soil behaviour in the *p*' - *q* space divided into two regions by the yield surface.

Elastic & plastic regions

Soil behaves purely elastically for any stress excursion inside the yield surface;





virgin yield occurs for stress state on the surface and causing it expansion.

During virgin yielding, yield surface expands with the current stress stays on the surface.



Mean effective stress lnp '



The compression behaviour of structured soils is described as

 $\mathbf{e} = \mathbf{e}^* + \Delta \mathbf{e}$

e*: voids ratio for the same soil in a reconstituted state

∆e: additional voids ratio sustained by soil structure

The compression behaviour of structured soils is described as

Based on experimental data, Liu and Carter (1999, 2000) proposed the following equation for Δe

$$\Delta e = a \left(\frac{p'_{y,i}}{p'} \right)^b + c$$

b: destructuring index;

c: the part of $\triangle e$ that cannot be eliminated by the increase of stress level; a: $\triangle e_i = a+c$.

Some basic assumptions are:

- (1) The mechanical properties of a clay in laboratory reconstituted states are treated as intrinsic, which can be described adequately by the MCC model.
 (2) Elastic properties of soil are independent of soil structure.
- (3) Both hardening and destructurig of soil are dependent on plastic volumetric deformation.

With the proposed isotropic compression line, the SCC model is formulated.

The Structured Cam Clay (SCC) model applied to simulate the behaviour of soil for (1) laboratory single element tests and (2) boundary value problems.

(1) Convenient identification of model parameters and for implementation into numerical analysis

(2) Successfully captures many important features of the behaviour of structured soils and influence of soil structure

(3) Significantly improves the performance of the Modified Cam Clay Model, represents well the behaviour of real soil, the soil found in nature

(4) Useful tool for the solution of boundary value problems encountered in geotechnical practice

Soil Plasticity and the Structured Cam Clay Model

III: Discussions on further development within the theoretical framework of the SCC model

- 1: Modeling soil deformation in the general stress and strain state
- 2: Plastic deformation within yield surface
- **3: Effect of cementation**
- 4: Development of soil structure
- 5: Post peak strength of soils
- 6: Anisotropy
- 7: Questions and discussions welcome!

In the previous talk,

(1)introduce plasticity theory for constitutive modelling of soils; Demonstrate how plastic deformation of soil modelled within the conventional plasticity theory with special consideration characteristics of soil.

A distinguished feature of soils: plastic volumetric deformation dependent hardening.

(2) introduce two elasto-plasticity models: original Cam Clay model and Structured Cam Clay model.

Models within the framework critical state soil mechanics

In SCC model, (1) differences between actual soil found in nature and that in laboratory reconstituted state illustrated. (2) The behaviour of natural soil modelled by considering the influence of soil structure

The SCC model formulated to be simple as a practical tool for geotechnical engineers. Simplifications and idealizations made in accordance with this requirement. Only features of first importance for common engineering problems represented.

Situations where a simple model not enough, refinement of the model or more advance models needed

Situations for examples

- (1) more accurate descriptions and detailed examination of soil behaviour
- e.g., features of second importance cannot be ignored.
- (2) more complicated material behaviour
- e.g., more complicated stress paths
- (3) special circumstances
- e.g., anisotropy significant

Improvement of the SCC model within the proposed theoretical framework possible

Some of techniques for the improvement discussed, they are

- 1: soil behaviour in the general stress and strain state
- 2: Plastic deformation within yield surface
- 3: Effect of cementation
- 4: Small strain behaviour
- **5: Anisotropy**

2: Soil behaviour in the general stress and strain state

Work needed to generalize the current 2-D SCC model, axysymmetical stress and strain circumstances, to the general stress and strain tensor state.

Examining soil behaviour in 2-D model, e.g., yield surface,

$$q^2 - M^{*2} p'(p'_s - p') = 0$$

$$\frac{d\varepsilon_d^p}{d\varepsilon_v^p} = \frac{2\eta}{M^{*2} - \eta^2}$$

2: Soil behaviour in the general stress and strain state

The key part to find stress and strain parameters suitable for representing soil behaviour in the stress and strain tensor space.

To substitute p', q, ε_v and ε_d .

The concept of a generalized shear stress ratio proposed as a mapping quantity to representing the mobilization of the friction resistance of soil in the general stress state. Then other stress and strain parameters formed in step with it.

A generalized shear stress ratio formulated based on a critical state strength criterion in the general stress space

Experimental data: (1) critical states of deformation exist for a wide range of geomaterials with and without structure;

(2) mechanical properties of soils at a critical state of deformation independent of soil structure and testing stress paths

Some experimental data

Failure surface of Fuji sand Detected in the 3-D principle Stresses (Yamada, 1979)

 π plane is used and defined as

$$x = \frac{\sqrt{2}}{2} (\sigma_2' - \sigma_3'), \quad y = \sqrt{\frac{3}{2}} (\sigma_1' - p')$$

The X & Y coordinates & conventional triaxial tests are shown



Fuji sand π **Plane**

Failure surface between von Mises failure surface & Matsuoka-Naki surface



Grundite clay (Lade et al) Failure surfaces of clay at different stress level



A rock (Mogi, 1971) Failure surface



2.1 A generalized shear stress ratio Strength criteria of geo-materials at critical state

(1)The criterion can be expressed in terms of the three stress invariants. Intrinsic material property, dependent of mineralogy of the material

 (2) Critical state surface in principal stress space is a linear cone, with the apex of the cone being at the origin of stress space if the material is cohesionless.
 The relative value of the stress no influence the shape of the surface 2.1 A generalized shear stress ratio Strength criteria of geo-materials at critical state in the general stress space determined by

Its shape in the π plane

Shape critical state Strength surface in the π plane varies between von Mises criterion and Matsuoka-Nakai criterion.

Liu and Carter (2003) proposed a general critical state Strength, allowing the shapes of the surfaces variable with materials

 $f_2 = \frac{(\sigma_1' + \sigma_2' + \sigma_3') \left[(\sigma_1' + \sigma_2' + \sigma_3')^2 - s \left(\sigma_1'^2 + \sigma_2'^2 + \sigma_3'^2 \right) \right]}{\sigma_1' \sigma_2' \sigma_3'} - 27 + 9s$



Fig. 1 Critical state strength surfaces in the π plane $(\phi_{cs}=32^\circ, p'=100 \text{ kPa})$

2.1 A generalized shear stress ratio A generalized shear stress ratio formulated

$$\eta^{*} = \frac{2 \left[f_{2} + \sqrt{f_{2}(f_{2} + 27 - 15s)} \right]}{f_{2} + 27 - 15s + \sqrt{f_{2}(f_{2} + 27 - 15s)}}$$



$$\eta^{\wedge} = \frac{2(\sigma_1' - \sigma_3')}{\sigma_1' + \sigma_3'}$$

For the stress on the surface with

$$\sigma_2' = \frac{(\sigma_1' + \sigma_3')}{2}$$

2.2 Stress parameters for general tensor stress state

Based on formulated generalized shear stress ratio Stress parameters for constitutive modeling for general stress and strain tensor proposed

$$p' = \frac{1}{3} (\sigma'_{11} + \sigma'_{22} + \sigma'_{33})$$
$$q^{\wedge} = p' \eta^{\wedge}$$

The generalized stress ratio is suitable for general tensor stress states.

2.3 Generalization of yield surface

2-D models can be extended for general stress states, for example,

Yield surface

$$q^2 - M^{*2} p'(p'_s - p') = 0$$

Rewritten as

$$q^{2}-M^{*2} p'(p'_{o}-p')=0$$

2.3 Generalization of yield surface

 σ'_3

Yield surface

$$q^{2} - M^{*2} p'(p'_{o} - p') = 0$$

The yield surface in the Principal stress space



 $\sigma'_1 = \sigma'_2 = \sigma'_3$

 σ'_2

2.3 Generalization of yield surface

Yield surface



2.4 Generalization of flow rule

Dilatancy for 2-D

$$\frac{d\varepsilon_d^p}{d\varepsilon_v^p} = \frac{2\eta}{\left|\mathbf{M}^{*2} - \eta^2\right| + \sigma\eta^2} \left|1 - \sqrt{\frac{p'_e}{p'_s}}\right|$$

Generalized dilatancy: by substituting the shear stress ratio

$$\frac{d\varepsilon_d^p}{d\varepsilon_v^p} = \frac{2\eta^n}{\left|\mathbf{M}^{*2} - \eta^{n^2}\right| + \varpi\eta^{n^2}} \left|1 - \sqrt{\frac{p'_e}{p'_s}}\right|$$
2.4 Generalization of flow rule

Generalized dilatany:

$$\frac{d\varepsilon_d^p}{d\varepsilon_v^p} = \frac{2\eta^n}{\left|\mathbf{M}^{*2} - \eta^{n^2}\right| + \omega\eta^{n^2}} \left|1 - \sqrt{\frac{p'_e}{p'_s}}\right|$$

Distortional strain increment obtained, not the increment plastic strain tensor.

Assumption

The deviatoric plastic strain increment tensor for loading is assumed to be linearly proportional to the current deviatoric stress tensor

2.4 Generalization of flow rule Mathematically

$$(d\varepsilon^p - d\varepsilon_v^p) = \Lambda(\sigma' - p')$$

The incremental plastic strain can be written as

$$d\varepsilon^{p} \neq \frac{\mathbf{I}}{3}d\varepsilon^{p}_{v} + \frac{(\mathbf{\sigma}' - p')\sqrt{6}d\varepsilon^{p}_{d}}{2\sqrt{(\mathbf{\sigma}' - p')} \cdot (\mathbf{\sigma}' - p')}$$

Unit tensor, stress ténsor, inner product of two tensors

The plastic deformation is defined.

3: Effect of cementation

Dr. Suksun Horpibulsuk has carried frontier research on the behaviour of cemented soil. I believe that he is more qualified than I to talk to you about the influence of cementation on the behaviour of soils and how to model these influence mathematically. I will not omit this topic here.

4: A conceptual framework for modelling the mechanical behaviour of structured soils

Soil behaviour very complicated and loading circumstances soil is subjected in geotechnical engineering vary greatly. The mechanisms for deformation and failure for geo-structures often controlled by different characteristics of soil behaviour. Comprehensive review of soil Behaviour necessary.

4: A conceptual framework for modelling the mechanical behaviour of structured soils

This section covers:

- Some well known characteristics of soil behaviour
- Quantification of the behaviour
- A conceptual framework for modelling

4: Conceptual Framework

- Useful:
 - guidance for mathematical modelling
 - understand which characteristics of soil behaviour which are most significant

4.1: Stable region

Experimental data Cyclic isotropic tests (EI-Sohby, 1964)



Cyclic isotropic loading of soil (El-Sohby, 1964)

Cyclic isotropic tests



Cyclic isotropic loading (El-Sohby, 1964) Soil behaviour during cyclic loading

(1)Virgin yielding for 1st cycle;

(2) stable after six cycles;

deformation dependent on type of loading and the stress state, independent of *N*.

(3) Transition from virgin yielding to stable behaviour for N = 2 ~5.

Cyclic tests on soils

Similar trends observed (a) for cyclic tests with η =constant; (b) for cyclic shearing tests if $\eta < \eta_s$. η_s , a given shear stress ratio.

A new region of soil deformation suggested: The stable region, usually covering the elastic region.

The stable region

In the region, soil deformation, independent of the number of cycles. No further accumulation in permanent deformation.

Deformation not elastic, hysteresis seen.

The region greater than elastic region.

Important for designs such as pavements, offshore foundations.

N for traffic infinitive

Pure elastic deformation within the yield surface assumption.

Seen from the original data, material idealization made. Unloading & reloading not the same, not completely recoverable not "exact" elastic = actual plastic



Plastic deformation occurs for loading inside yield surface. **Concept of subyielding** introduced. **Destructuring**, associated with plastic deformation, occurs in subyielding.



importance

yielding.

(a) Loading with large stress reversals

For loading, virgin yielding Unloading, at start, elastic As reversal , plastic occurs At large reversal, plastic defm. is much greater than virgin



The behaviour of Beaucaire clay under isotropic loading and unloading tests (Costanzo et al, 2006)

(b) Liquefaction,

The feature of plastic deform.

similar to the isotropic unloading. Huge plastic deformation occurs as the stress goes to zero.

If subyielding not considered, no liquefaction



(c) Structure performance under cyclic loading

excavation, earthquake, wave, traffic loading

(d) For particularly structures, allowed deformation or non-uniform deformation small, accurate prediction of soil deformation needed.

An important one, non-uniform settlement of building caused by tunnelling in metropolitan cities.

4.3: Development of soil structure with time

- Soil forms and develops structure with time.
- Usually positive effect.
- The peak strength increases;
- The stiffness increases.



Example: an enlargement of the yield stress after a few months

4.3: Development of soil structure with time

- Drained sheared test on a sand
- The stiffness increases significantly



FIG. 10. Laboratory Example of Modulus of Sand Increasing with Secondary Compression Aging

4.4: Different soil structures due to the soil formation proc 80 Syncrude sand $\sigma'_{vc} = 200 \text{ kPa}$ **Undrained test** σ_{v} 60 Vaid et al (1995) The same a sand with 40 different methods of

sample preparation Basically the same

voids ratio



Fig. 19. - Effect of specimen reconstitution method on undrained simple shear response (after Vaid et al., 1995)

4.4: Different soil structures due to the soil formation



Fig. 27. Undrained behaviour of medium loose sample of Ishihara (1993) Toyoura sand: moisture placement

4.4: Different soil structures due to the soil formation p



Ishihara (1993) Toyoura sand: dry deposition placement 4.5: Post peak strength of soils Three strengths for soils (a) peak strength (b) critical state strength (c) residual strength The shear strength of a soil that can be mobilised on a polished sliding surface, after it has been formed through the soil due to the alignment of its platy particles.

4.5: Post peak strength of soils

For residual strength

(1) enough platy particle to form polished failure surface

(2) large deformation to form polished surface

Critical state strength for most clay: 20°

Residual strength: as low as 6°

Landslides can be controlled by residual strength, especial slide along pre-existing failure surface

Critical state strength and residual strength dependent on clay faction ω_c

Influence in three regions (1) $\omega_c < 20\%$, no influence CS Strength = R strength constant

(2) $\omega_c > 50\%$, no influence CS Strength > R strength both constant

(2) 20%< ω_c < 50%, no influence CS Strength > R strength both decreases with ω_c



4.6 Influence of soil anisotropy The greatest challenges to academia and engineering in engineering

Some influences demonstrated and discussed Isotropy and anisotropy

A material is defined as isotropy if it possesses no preferred direction; and the orientation in space of a sphere of an isotropic material can not be detected experimentally.

Otherwise, the material is anisotropy

Anisotropy Isotropy and anisotropy

A material is defined as isotropy if it possesses no preferred direction; and the orientation in space of a sphere of an isotropic material can not be detected experimentally. Otherwise, the material is anisotropy



Anisotropy **Cross anisotropy Formulation and property** Most geo-materials found in site cross anisotropic arising from the depositional history of natural soil



Anisotropy: test data

- Isotropic compression tests on sand
- σ'1= σ'2= σ'3=*p*'
- → ε1=ε2=ε3
- ε**ν/** ε**a=3**
- Virgin compression n=37.5%
- ≈ isotropy
- Anisotropy increases with n.



Fig I.2-1 Axial strain and volumetric strain during cyclic isotropic loading (after El-Sohby 1964)

Anisotropy: test data **Isotropic compression tests** σ'1 = σ'2 = σ'3 = p' $\varepsilon v/\varepsilon a=3$ Unloading The same $\varepsilon v/\varepsilon a=3$ **Elastic & isotropic for** this sand



Fig I.2-1 Axial strain and volumetric strain during cyclic isotropic loading (after El-Sohby 1964)

Anisotropy:

Undrained effective stress paths

Fig. A4. Undrained effective stress paths for different combinations of b and $\alpha = 0^{\circ}$ at OCR = 1.0.



Anisotropy:

Undrained effective stress paths Indicating rotation of the yield surface

Fig. A4. Undrained effective stress paths for different combinations of b and $\alpha = 0^{\circ}$ at OCR = 1.0.



Anisotropy: test data



Yield surface identified for some natural clays: rotated toward ko line

Anisotropy: test data

Anisotropy linked to effect of the principal stress rotation

The same as cutting samples from different directions & testing them in the same way

If soil isotropic, response the same; If soil anisotropic, response

dependent on angle θ



ground

Rotation of the principal stresses

Hollow cylinder test Toyoura sand different directions of the principal stresses

Behaviour of soil dependent on angle θ: direction of principal stress → anisotropic



Anisotropy: test data

Peak strength direction of the principal stresses

For most sand, minimum peak strength at $\theta=20^{\circ}\sim30^{\circ}$



Anisotropy: test data

Effect of rotation of principal stresses

In tests, σ'_{11} , τ_{12} , σ'_{22} , τ_{21} vary in such as way that magnitudes of the two principal stresses σ'_1 and σ'_2 do not change but the directions of σ'_1 and σ'_2 changes continuously from 0° to 360° to 720°, and on.



Effect of rotation of principal stresses

 $E2 = \varepsilon_{11} - \varepsilon_{22}$ **E4=ε12** In rotation of the principal stresses: (1) plastic deformation (2) permanent strain increases



Fig I.2-27 Soil behaviour for cyclic rotation of the principal stresses with fixed magnitudes (after Alawi 1989)
Conceptual Framework

- Four regions of different behaviour
 - Elastic region
 - Virgin yielding
 - Sub-yielding
 - "Stable" behaviour



5 Surfaces in Stress Space

- Structural yield surface
- Equivalent yield surface
- Loading surface
- Elastic surface
- Stable surface



Behaviour Elastic Condition Stress state inside the elastic surface, p'_{e}

Elastic Response



BehaviourConditionElasticStress state inside the elastic
surface, p'_e Virgin yielding $p'_c = p'_s$ and $dp'_c > 0$

Virgin Yielding



BehaviourConditionElasticStress state inside the elasticsurface, p'_e Virgin yielding $p'_c = p'_s$ and $dp'_c > 0$ SubyieldingStress state inside p'_s and out side p'_e and p'_n

Sub-Yielding



BehaviourConditionElasticStress staaurface, pVirgin yielding $p'_c = p'_s$ arSubyieldingStress sta p'_e and p'_r SubyieldingStress sta

Stress state inside the elastic surface, p'_e $p'_c = p'_s$ and $dp'_c > 0$ Stress state inside p'_s and out side p'_e and p'_n Stress state inside p'_n and out side p'_e during the first few cycles

Sub-Yielding (1st few cycles)



Condition

Behaviour

Stable Response (cycling)



Loading Type Condition First loading $p'_c = p'_{cmax}$ and $dp'_c > 0$

p'_{cmax} is the maximum loading surface the soil has ever experienced.

Loading TypeConditionFirst loading $p'_c = p'_{cmax}$ and $dp'_c > 0$ Reloading $p'_c < p'_{cmax}$ and $dp'_c > 0$

p'_{cmax} is the maximum loading surface the soil has ever experienced.

Loading TypeConditionFirst loading $p'_c = p'_{cmax}$ and $dp'_c > 0$ Reloading $p'_c < p'_{cmax}$ and $dp'_c > 0$ Unloading $p'_c < p'_{cmax}$ and $dp'_c < 0$

p'_{cmax} is the maximum loading surface the soil has ever experienced.

Hardening & Destructuring

 Assumed to depend on the plastic volumetric deformation

Consequences

- Magnitude of plastic volumetric deformation of natural soil is dependent on change in size of yield surface, irrespective of stress path
- Structural yield surface is dependent on current soil structure, current voids ratio, and current stress state

Changes in Structure

- Destructuring
 - caused by stress changes
 - monotonic and irrecoverable
- Development of structure
 - all other effects such as ageing, leaching, change of chemical components of the pore fluid, and weathering



(b) Variation of surfaces during loading

Destructuring

Sub-yielding: AB



Destructuring

Sub-yielding: AB Virgin yielding: BC





(b) Variation of surfaces during loading

Destructuring

Sub-yielding: AB **Virgin yielding: BC** Unload. & reload.: CDE

Development of Structure

 Many factors affect development of structure: e.g., ageing effect on the size of the structural yield surface (Mesri and Shahien, 1997)

$$p_{y,i}' = p_o' \left(\frac{t}{t_p}\right)^{\beta}$$

- p'o size of equivalent yield surface
- t_p time needed for primary compression
- time measured from end of primary compression
- β material constant.

Soil Strength

- Final state under shearing is either the critical state of deformation or the residual state of deformation
- Clay fraction < 20% critical state strength
- Clay fraction > 20%

continuous strength reduction after critical state requires description

Strength Reduction

 Strength after the critical state depends on particle orientation

 $M_{f} = M_{cs}^{*} - (M_{cs}^{*} - M_{r}^{*})\omega_{o}$

M_f maximum shearing resistance
M*_{cs} critical state strength
M*_r residual strength
ω_o degree of particle orientation
(linked with the plastic work of shear
deformation along potential sliding surface)

Summary

7 Major points

- 1. Equivalent yield surface, structural yield surface and the loading surface
- 2. Elastic surface
- 3. Stable surface

Summary

- 1. Equivalent yield surface, structural yield surface and the loading surface
- 2. Elastic surface
- 3. Stable surface
- 4. Flow rule
- 5. Hardening and destructuring
- 6. Stable deformation
- 7. Transition to the residual state of deformation

Conclusions

- Conceptual framework describing mechanical behaviour of structured soils
- Stress-strain behaviour divided into four regions in stress space, *i.e.*,
 - an elastic region, a stable deformation region, a sub-yielding region and a virgin yielding region
- Influence of structure on mechanical behaviour discussed

Thank you very much!