Soil Plasticity and the Structured Cam Clay Model

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Soil Plasticity and the Structured Cam Clay Model

Summary
I: Soil plasticity
II: The Structured Cam Clay model
III: Further development within the theoretical framework of the SCC model
Soil Plasticity and the Structured Cam Clay Model

I: Main ideas introduced in Soil plasticity

Constitutive model:
-describing the change in the strain state of an element of material to the change in the stress state acting on the element.

Our knowledge of soil mechanics comes from human practice. And our research on soil mechanics is to help human practice. The soil mechanics from engineering practice and for engineering practice is a life theory.

Effective stress principle: foundation stone of modern soil mechanics.

⇒ Principles of continuum mechanics can be applied to soils.
Soil Plasticity and the Structured Cam Clay Model

I: Main ideas introduced in Soil plasticity

Elastic deformation: recoverable after the removal of force, and independent of stress path.

Modelled by Hook’s law.

When the response of a soil to loading is dependent on the direction of loading, the soil is anisotropic.

Plastic deformation: irrecoverable, and dependent on stress path

Three components of soil plasticity:
(i) Yield surface, (ii) Flow rule and (iii) Hardening function
I: Main ideas introduced in Soil plasticity

(i) Yield surface in the stress space

\[ f(p', q, p'_o) = 0 \]

Size of the yield surface

dividing soil behaviour into two regions:
loading inside the surface: elastic;
loading on the surface, plastic.

(ii) Flow rule: the direction of plastic strain increment

Plastic strain flow rule: independent of the stress increment.

\[ \frac{d \varepsilon^p_v}{d \varepsilon^p_d} = r(p', q) \]

Direction of plastic strain increment
normal to the yield surface:
associated flow rule.
NOT: non-associated flow rule.
I: Main ideas introduced in Soil plasticity

(iii) Hardening of the yield surface
Variation of yield surface with plastic deformation.

For models of the Cam Clay family, hardening of yield surface: plastic volumetric deformation dependent

\[ d\varepsilon_v^p = h(p', q)dp'_o \]

With these three parts defined, plastic deformation found. From (i), the size change of the yield surface obtained as

\[ dp'_o = -\frac{\partial f}{\partial p'_o} dp' + \frac{\partial f}{\partial q} dq \]
I: Main ideas introduced in Soil plasticity

Modelling plastic deformation

(i) Yield surface:
\[ f(p', q, p'_o) = 0 \]

(ii) Flow rule:
\[ \frac{d \varepsilon^p_v}{d \varepsilon^p_d} = r(p', q) \]

(iii) Hardening of the yield surface
\[ d \varepsilon^p_v = h(p', q)dp'_o \]

Then plastic strain increment can be expressed as

\[
\begin{align*}
\left\{ 
\begin{aligned}
d \varepsilon^p_v &= -h(p', q) \times \frac{\partial f}{\partial p'} dp' + \frac{\partial f}{\partial q} dq \\
\quad &+ \frac{\partial f}{\partial p'_o} dp'_o \\

&= -h(p', q)r(p', q) \times \frac{\partial f}{\partial p'} dp' + \frac{\partial f}{\partial q} dq \\
\quad &+ \frac{\partial f}{\partial p'_o} dp'_o 
\end{aligned}
\right.
\end{align*}
\]
Soil Plasticity and the Structured Cam Clay Model

I: Soil plasticity

II: The Structured Cam Clay model

1: Introduction
2: Influence of soil structure
3: Study on compression behaviour of structured clay
4: Formulation of Structured Cam Clay model
5: Validation of the model
6: Application of the model
7: Questions and discussions at any time welcome!
1: Introduction

Since the formulation of the Cam Clay model, plasticity has found wide application in geotechnical engineering. NOW, constitutive modelling of soil is a fashion. Now very necessary fashion.

Why?

(1) Essential to see the details of the performance of structures;
   Virtually no computation can be performed except the strength.

(2) A key factor to control the accuracy of numerical analysis (BEM, FEM).
Current situation:
Hundreds of models developed by academia in their office. Almost all the models developed from laboratory observations on reconstituted soil. Virtually all models geotechnical practitioners use in their very advanced numerical analysis package are the models that have been labelled for reconstituted soils.

Question needed to be raised: natural soil = reconstituted?
Natural soil \(\leftrightarrow\) reconstituted soil

Natural soil: the soil you find in nature.
reconstituted soil: the soil we play with in the school.
(i) Taking from a special place on planet;
(ii) drying and smashing it to powder;
(iii) Mixing the soil powder with water. \(\Rightarrow\) we make a new soil

Is the behaviour of a soil in nature the same as that of the same soil reconstituted in laboratory

Before trying answer, should we perform some investigation?
2: Influence of soil structure: Experimental evidence

(1) Oedometer test on Mexico City Clay
    Terzaghi (1953).
    Oedometer test (1-D test): loaded vertically with no lateral deformation.

Two tests: a nature intact soil sample and a reconstituted sample.

A nature intact soil:
    Taken carefully from site with minimum disturbance. Soil supposed to be like its original state in the field.

A reconstituted soil:
    Soil taken from site, dried and smashed to powder, then mixed with distilled water thoroughly.
2: Influence of soil structure: Experimental evidence

Conclusion:
Compression behaviour remarkably different!

Mexico city clay extremely high voids and extremely compressible

Quantitative analysis

Oedometer test on Mexico City Clay (Terzaghi, 1953)
Oedometer test on Mexico City Clay
(Terzaghi, 1953)

Soil behaviour
At $p'=100$ kPa

Voids ratio $e$
A : $e=11$
B: $e=7.4$
$\Delta e=11-7.4=3.6$

Compression index $\lambda$
A: $\lambda=14.5$
B: $\lambda=1.65$
$\lambda(A)/\lambda(B)=9$

9 times more compressible
Using reconstituted soil to represent natural soil?

Stiffness:
For $p' < 100$ kPa, over-prediction of compressive deform.;
For $p' > 100$ kPa, seriously under-prediction of deform..

Voids ratio
Always under-estimated

In most model, strength dependent on voids ratio ➔ leading to over-prediction of the strength

Oedometer test on Mexico City Clay (Terzaghi, 1953)
(1) Oedometer test on Mexico City Clay, Terzaghi (1953).

(2) Undrained unconfined triaxial test on London Clay (Skempton, 1952).
A nature intact London clay & a reconstituted London clay:
For both samples:
soil mineralogy,
initial voids ratio $e$,
initial stress state $\sigma'$,
testing stress path

\[ \Rightarrow \text{The same response?} \]

Shear stress $q$: $q = \sigma'_1 - \sigma'_3$.

\[ \Rightarrow \text{ALL the same} \]

By Cam Clay model, yes!
Conclusion: 
Shearing behaviour completely different!

Behaviour pattern
Reconstituted soil:
Hardening steadily until failure reached; only one strength, the ultimate failure strength.

Natural soil:
Initially hardening and reaching a peak strength, finally softening failure. Two strength: peak strength & final failure strength.

Undrained test on London Clay (Skempton & Nouthry, 1952)
(2) Shearing behaviour

Undrained test on London Clay (Skempton & Northry, 1952)

**Strength**
- Recon.: $q_{peak} = 4.6$ kPa
- Natural: $q_{peak} = 37.3$ kPa

$$\frac{q_{peak}}{\text{natural}} = \frac{37.3}{4.6} = 8$$

**Stiffness at $\varepsilon_d = 5\%$**
- Recon.: $E = 64$ kPa
- Natural: $E = 750$ kPa

$$\frac{E}{\text{natural}} = \frac{750}{64} = 12$$
2: Influence of soil structure: Experimental evidence

(1) Oedometer test on Mexico City Clay, Terzaghi (1953).

(2) Undrained unconfined test on London Clay, (Skempton, 1952).

Conclusion:
(i) Behaviour of nature intact clay & that of the same clay in a reconstituted state are not the same.

(ii) Models, developed for laboratory reconstituted soil, are applied for soil in situ, significant errors can occur.

(iii) Mineralogy of soil not enough for defining mechanical properties of soil, the soil structure factor must be added into constitutive modelling of soil.
Soil structure

**Soil structure**: arrangement and bonding of soil particles.

When reconstituted, the original structure of the soil is destroyed.

In this study, difference in behaviour between a soil found in nature and that reconstituted in laboratory.

**Formation**: formed during depositional and geological histories. Factors: ageing, chemical reaction, temperature effect, weathering, loading etc.

Complicated & difficult to trace.

Quantifying the formation of soil structure impossible such the exact histories of the soil and environments
Some basic assumptions and definitions

**Destructuring:** removal of soil structure.

**Intrinsic properties:** independent of soil structure, denoted (*), such $\lambda^*$, $\kappa^*$.

Mechanical properties of reconstituted soil, intrinsic.

**Influence of Soil Structure:** difference in behaviour between structured soil and the soil in reconstituted status.

**Assumption:** If soil structure completely removed, natural soil = reconstituted soil.

Critical State of deformation: soil being sheared continuously $\Rightarrow$ soil structure completely removed.

$\Rightarrow$ Mechanical properties of soil at critical states independent of soil structure, intrinsic.
3: Study on compression behaviour of natural clays

Compression behaviour of natural soils examined compared with that of the reconstituted soil.

A material idealization of the compression behaviour
Equation for the compression behaviour of structured soil

These results used in the formulation of the SCC model.

Three sets of data shown
3: Study on compression behaviour of natural clays

Oedometer test on Mexico City Clay by Terzaghi (1953)
1 D compression

![Voids ratio e vs Mean effective stress $p'$ (kPa)](image)

- **Natural soil**
- **Reconstituted**
Compression behaviour of natural soils

(2) Stiff Pleistocene clay
by Cotecchia (1996)
Cyclic tests performed

For virgin yielding, behaviour very different

For unloading & reloading, no much difference between a reconstituted soil and a natural soil approximately parallel

One-dimensional compression tests on stiff Pleistocene clay
3: Compression behaviour of natural soils

(3) Compression behaviour of soft Nagasaka clay performed by Murakami, (1979)
   Two tests: Both on reconstituted Nagasaka clay
   One: conventional compression test
   Other: soil sample is loaded to given stress level, then stopped to allow creep to occur. After a few months, soil is tested again.
3: Compression behaviour of natural soils

Soft Nagasaka clay
Murakami, (1979)

Sample one: conventional compression test
Soil: reconstituted
Behaviour linear $e - \ln p'$

Typical behaviour of reconstituted clay

Compression behaviour of soft Nagasaka clay (Murakami, 1979)
3: Compression behaviour of natural soils

Soft Nagasaka clay
Murakami, (1979)

Sample two:
Holding for some month: creep & ageing

Soil structure:
Developed over time
(i) Yield at stress much higher than $p'_\text{max}$.
(ii) Non-linear $e - \ln p'$

Compression behaviour of soft Nagasaka clay (Murakami, 1979)
3: Compression behaviour of natural soils

Soft Nagasaka clay
Murakami, (1979)

(iii) During yielding, difference between reconstituted and natural soils narrows

⇒ Destructuring (structure removed)

(iv) When \( p' \) large,

⇒ Structure of soil completely removed

⇒ the two soil behave identically.
Compression behaviour of structured clays:

(1) For a given stress, $p'$, the voids ratio for a natural, structured soil is higher than that of the same soil the reconstituted soil of. The higher the difference in voids ratio, the stronger the soil structure.

(2) When soil undergoes plastic deformation, destructuring occurs and the additional voids ratio sustained by soil structure decreases.
3: Compression behaviour of natural soils

Compression behaviour of structured clays:

(1) For a given stress, $p'$, the voids ratio for a natural, structured soil is higher than that of the same soil the reconstituted soil of. The higher the difference in voids ratio, the stronger the soil structure.

(2) When soil undergoes plastic deformation, destructuring occurs and the additional voids ratio sustained by soil structure decreases.

(3) As $p'$ increases, the compression curves corresponding to the structured soils appear to be asymptotic to the curve for the reconstituted soil, i.e. the influence of soil structure tends to diminish as $p'$ increases.
A material idealisation of the compression behaviour of structured soils

Following the Cam Clay model:
Compression behaviour of reconstituted soil, \( e^* \), linear in the \( e - \ln p' \) space, ICL*

Compression behaviour of structured soil,
Above the reconstituted soil
\( e \): structured soil.

Structured soil
\( e = e + \Delta e \)
A material idealisation of the compression behaviour of structured soils

\[ \Delta e = e - e^* \]

the difference, the additional voids ratio sustained by soil structure.

\[ \Delta e = e - e^* \]

\[ e = e + \Delta e \]

\[ \Delta e \]: the influence of soil structure.

\[ \Delta e = 0 \], soil structure has no effect.
A material idealisation of the compression behaviour of structured soils

\[ e = e^* + \Delta e \]

\( \Delta e \): additional voids ratio sustained by soil structure

Based on the work by Liu and Carter (1999, 2000)

\[ \Delta e = a \left( \frac{p'_{y,i}}{p'} \right)^b + c \quad \text{for} \ p' \geq p'_{y,i} \]
A material idealisation of the compression behaviour of structured soils

\[ \Delta e = a \left( \frac{p'_{y,i}}{p'} \right)^b + c \quad \text{for } p' \geq p'_{y,i} \]

\( p'_{y,i} \): initial yield stress associated with soil structure

\( \Delta e_i \): initial additional voids ratio at \( p' = p'_{y,i} \).

\( b \): destructuring index;

\( c \): the part of \( \Delta e \) that cannot be eliminated by the increase of stress level;

\( a \): \( \Delta e_i = a + c \).
\[ \Delta e = a \left( \frac{p'_{y,i}}{p'} \right)^b + c \]

For natural soft clay:
Usually assumed
\( b = 1 \)
\( c = 0 \), additional voids ratio completely diminished.
Aim: a simple predictive model for the solution of practical geotechnical problems, which can provide reasonable prediction of the behaviour of natural clays for loading within the valid range of the model.

Necessarily to keep the model simple and convenient for geotechnical practitioners to solve practical problem
The Structured Cam Clay (SCC) model formulated by introducing the effect of soil structure to Modified Cam Clay (MCC) model.

The MCC model: the base for the new model. **Reason:**

(1) Simple and elegant, yet with clear physical and thermodynamics ground.
(2) Description of the behaviour of reconstituted soil with acceptable accuracy.
(3) Most popular and widely applied in geo-engineering field.
(Perhaps the only one used for design purpose).
4: Formulation of Structured Cam Clay model

4.1: Basic ideas:

(1) Properties of a clay in laboratory reconstituted states are treated as intrinsic. Independent of soil structure. The intrinsic properties used as a standard to measure the influence of soil structure.

The difference in behaviour between an intact soil and the same soil in a reconstituted state \( \Rightarrow \) influence of soil structure.
4: Formulation of Structured Cam Clay model

(2) It is assumed that the behaviour of a soil in a reconstituted state is adequately described by Modified Cam Clay (MCC) model.

In formulating SCC model, the difference in soil behaviour is of interest, the focus.
In most models of the Cam Clay family, soil response defined in a 3-D space:
   (a) \( e \): current voids ratio,
   (b) \((p', q)\): Current stress state,
   (c) stress history.
For soil of a given mineralogy, the response of the soil to a given loading is the same if all the three elements are the same.

(3) For Structured Cam Clay model, soil response defined in a 4-D space:
   (a) \( e \): current voids ratio,
   (b) \((p', q)\): Current stress state,
   (C) stress history.
   (d) soil structure
4: Formulation of Structured Cam Clay model

Basic ideas:

(4) Only the isotropic variation of the soil mechanical properties associated with soil structure are modelled.

Anisotropy normally develops with soil structure. For simplicity, not considered in the SCC model.

Formulation of SCC model includes
(1) Material idealization,
(2) Elastic deformation,
(3) Plastic volumetric deformation during virgin yielding,
(4) Plastic shear deformation during virgin yielding.
4.2 Material Idealisation
Yield surface for structural clays

Collection of all the yield stress points in the $p'$-$q$ space forms the yield surface.
Yield surface for structural clays

Yield surface for reconstituted soil: dependent only on stress history;

Yield surface for structured soil: dependent on soil structure and stress history;
4.2 Material Idealisation

Effect of soil structure on yield surface

(1) Isotropic variation of the surface
Modelled by $p'_s$, the size of the structural yield surface.

(2) Anisotropic variation of the surface
Distortion of the surface: not considered
⇒ no distortion on the surface
4.2 Material Idealisation

Structural yield surface:
the same as that of the MCC model. Elliptical in stress space $p'-q$; the aspect ratio $M^*$, the critical state shear strength

$$q^2 - M^{*2} \left(p' \left(p'_s - p'\right)\right) = 0$$
4.2 Material Idealisation

Soil: idealised as an elastic and virgin yielding material.

Two zones of soil behaviour by the yield surface

(1) Pure elastic behaviour: loading inside the yield surface

No plastic deformation, yield surface remains unchanged.
(2) Virgin yielding behaviour:

For stress on the yield surface and causing its expansion.

Plastic deformation, yield surface expansion, stress staying on the yield surface.
4.3 Elastic deformation

Two basic assumptions

(1) Soil obeys Hook’s law.

(2) Elastic properties of soil are independent of soil structure.

\[
d\varepsilon^e_v = \frac{3 (1 - 2\nu^*)}{E^*} dp'
\]

\[
d\varepsilon^e_d = \frac{2 (1 + \nu^*)}{3} dq
\]

\[E^*\]: Young’s modulus; \[\nu^*\]: Poisson’s ratio;
\[*\]: intrinsic properties
4.3 Elastic deformation

\[
d\varepsilon_v^e = \frac{3(1 - 2\nu^*)}{E^*} dp'
\]

\[
d\varepsilon_d^e = \frac{2(1+\nu^*)}{3} \frac{dq}{E^*}
\]

Young’s modulus \(E^*\), Poisson’s ratio \(\nu^*\) and swelling index \(\kappa^*\) are related by

\[
E^* = \frac{3(1 - 2\nu^*)(1+e)}{\kappa^*} p'
\]
4.4 Plastic volumetric deformation during virgin yielding

Fundamental assumption of SCC model:

Both hardening and destructurig of soil are dependent on plastic volumetric deformation.

Foundation stone are the same for MCC and SCC.
4.4 Plastic volumetric deformation during virgin yielding

Let’s examine isotropic compression behaviour

\[ e = e^* + \Delta e \]

\( \Delta e \): additional voids ratio sustained by soil structure

Based on our research

\[ \Delta e = a \left( \frac{p'_{y,i}}{p'} \right)^b + c \]
4.4 Plastic volumetric deformation during virgin yielding

Isotropic compression

\[ e = e^* + \Delta e \]

Additional voids ratio \( \Delta e \)

\[ \Delta e = a \left( \frac{p_{y,i}'}{p'} \right)^b + c \]

\( e^* \): voids ratio for reconstituted clays: intrinsic material behaviour, modelled by MCC model

\[ e^* = e^*_{IC} - \kappa^* \ln p' - (\lambda^* - \kappa^*) \ln p' \]
4.4 Plastic volumetric deformation during virgin yielding

The Isotropic compression equation for a structure clay

\[ e = e_{IC}^* - \kappa^* \ln p' - (\lambda^* - \kappa^*) \ln p' + a \left( \frac{p_{y,i}'}{p'} \right)^b + c \]
4.4 Plastic volumetric deformation during virgin yielding

The Isotropic compression behaviour of a structure soil

**Assumption**: Elastic properties of soil are independent of soil structure

⇒ elastic deformation of structured clay = that of a reconstituted clay

\[
e = e_{IC}^* - \kappa^* \ln p' - (\lambda^* - \kappa^*) \ln p' + a \left( \frac{p'_{y,i}}{p'} \right)^b + c
\]

The overall elastic deformation for structured clay
4.4 Plastic volumetric deformation during virgin yielding

The Isotropic compression behaviour of a structure soil

Based on theory of MCC model

Plastic deformation for reconstituted clay

\[ e = e^*_{IC} - \kappa^* \ln p' - (\lambda^* - \kappa^*) \ln p' + a \left( \frac{p'_{y,i}}{p'} \right)^b + c \]
4.4 Plastic volumetric deformation during virgin yielding

Deformation associated with the additional voids, influence of soil structure, plastic deformation

Plastic deformation representing the influence of soil structure

\[ e = e^*_{IC} - \kappa * \ln p' - (\lambda^* - \kappa^*) \ln p' + a \left( \frac{p'_{y,i}}{p'} \right)^b + c \]
4.4 Plastic volumetric deformation during virgin yielding

The Isotropic compression behaviour of a structure soil

\[ e = e^{*}_{IC} - \kappa^{*} \ln p' - (\lambda^{*} - \kappa^{*}) \ln p' + a \left( \frac{p'_{y,i}}{p'} \right)^b + c \]

- **Constant, no contribution to deformation**
- **Elastic deformation**
- **Plastic deformation for reconstituted clay**
- **Plastic deformation for soil structure**
4.4 Plastic volumetric deformation during virgin yielding

Fundamental assumption of SCC model:

Both hardening and destructurig of soil are dependent on plastic volumetric deformation.

Size change of the yield surface dependent on plastic deformation only, irrespective of the stress path.

Plastic volumetric deformation written in terms of the Size change of the yield surface as

\[ d\varepsilon_v^p = f(p'_s)dp'_s \]

Independent of the stress path
4.4 Plastic volumetric deformation during virgin yielding

Additional voids ratio is associated with plastic volumetric deformation, therefore described in terms of the size of the structural yield surface, $p'_s$, not the stress state.

Mathematical format for $\Delta e$ in general stress states

$$\Delta e^p = \Delta e^p(p'_s)$$
In the isotropic compression equation

\[ e = e^{*}_{IC} - \kappa^{*} \ln p' - (\lambda^{*} - \kappa^{*}) \ln p' + a \left( \frac{p'_{y,i}}{p'} \right)^b + c \]

\( \Delta e \), as well as \((l^{*} - k^{*}) \ln p'\), are plastic deformation, their variation is dependent on the size change of the yield surface, should be written in terms of the size of the structural yield surface, \( p'_s \).
In the isotropic compression equation

\[ e = e_{IC}^* - \kappa^* \ln p' - (\lambda^* - \kappa^*) \ln p' + a \left( \frac{p_{y,i}'}{p'} \right)^b + c \]

For isotropic loading and for soil with elliptical yield surface, \( p'_s = p' \).

Change all parts for plastic deformation by \( p'_s \), we obtain the compression equation for loading along general stress path

\[ e = e_{IC}^* - \kappa^* \ln p' - (\lambda^* - \kappa^*) \ln p'_s + a \left( \frac{p_{y,i}'}{p'_s} \right)^b + c \]
4.4 Plastic volumetric deformation during virgin yielding

Differentiating the equation, dividing by \((1+e)\) and with some manipulation

\[
\frac{de}{1+e} = \frac{\kappa^*}{1+e} \left( \frac{dp'}{p'} \right) + \left( \lambda^* - \kappa^* \right) + b(\Delta e - c) \frac{dp'}{(1+e)p'_s}
\]

\[d\varepsilon_v\]

Considering when additional voids ratio completely removed, soil behaves the same as a reconstituted soil

\[
d\varepsilon_v = \frac{\kappa^*}{1+e} \left( \frac{dp'}{p'} \right) + \left( \lambda^* - \kappa^* \right) + b < \Delta e - c > \frac{dp'_s}{(1+e)p'_s}
\]

where

\[
\langle a \rangle = \begin{cases} 
  a & \text{if } a \geq 0 \\
  0 & \text{if } a < 0 
\end{cases}
\]
Further modification

Observed: the effect of shearing on the destructuring.

Modification made on plastic volumetric deformation to include the effect of shearing on destructuring. Finally,

\[ d\varepsilon_v = \frac{\kappa^* dp'}{(1+e)p'} + (\lambda^* - \kappa^*) \frac{dp_s'}{(1+e)p_s'} + \]

\[
\left[ \langle \Delta e - c \rangle + \frac{\gamma \eta \Delta e}{M^* - \eta} \right] \frac{b dp_s'}{(1+e)p_s'}
\]

Effect of shearing on destructuring
Volumetric deformation for loading along general stress path

The deformation made up of three parts:

\[ d\varepsilon_v = \frac{\kappa \star dp'}{(1+e)p'} + \left( \lambda \star -\kappa \star \right) \frac{dp'_s}{(1+e)p'_s} + \left[ \langle \Delta e - c \rangle + \frac{\gamma \eta \Delta e}{M \star -\eta} \right] \frac{b_dp'_s}{(1+e)p'_s} \]

Elastic deformation: \( dp' \)
The same as by Hook’s law
Volumetric deformation for loading along general stress path

Elastic deformation
The same as by Hook’s law

\[ d\varepsilon_v = \frac{\kappa^* dp'}{(1+e)p'} + \left(\lambda^* - \kappa^*\right) \frac{dp'_s}{(1+e)p'_s} + \left(\Delta e - c\right) + \frac{\gamma \eta \Delta e}{M^* - \eta} \frac{bp'_s}{(1+e)p'_s} \]

Plastic deformation
The same as described by MCC model deformation associated with the intrinsic properties of soil

Additional of SCC model: plastic deformation associated with the influence of soil structure
4.5 Plastic shear deformation during virgin yielding

Flow rule

MCC model: associated flow rule, the plastic strain increment is given by

\[
\frac{d\varepsilon^p_{d}}{d\varepsilon^p_{v}} = \frac{2\eta}{M^*^2 - \eta^2}
\]

Structured clays, observed that soil is more compressible. The higher the value of \(\Delta e\), the higher the compressibility. 

\(===>\) modification

\[
\frac{d\varepsilon^p_{d}}{d\varepsilon^p_{v}} = \frac{2\eta}{\left|M^*^2 - \eta^2\right| + \alpha \left|1 - \sqrt{\frac{p'_e}{p'_s}}\right|}
\]
4.5 Plastic shear deformation during virgin yielding

Flow rule

\[
\frac{d\varepsilon^p_d}{d\varepsilon^p_v} = \frac{2\eta}{\left| M \ast^2 - \eta^2 \right| + \sigma \left[ 1 - \sqrt{\frac{p_e'}{p_s'}} \right]}
\]

Plastic shear strain increment

\[
d\varepsilon^p_d = \frac{2\eta}{\left| M \ast^2 - \eta^2 \right| + \sigma \left[ 1 - \sqrt{\frac{p_e'}{p_s'}} \right]} \times \left\{ (\lambda \ast - \kappa \ast) + b \left[ \langle \Delta e - c \rangle + \frac{\gamma \eta \Delta e}{M \ast - \eta} \right] \right\} \frac{dp_s'}{(1 + e)p_s'}
\]
4.6 Incremental stress and strain relationship

\[ d\varepsilon_v = d\varepsilon^e_v + \left( \lambda^* - \kappa^* \right) \frac{dp'_s}{(1+e)p'_s} + \left[ \langle \Delta e - c \rangle + \frac{\gamma\eta\Delta e}{M^* - \eta} \right] \frac{bdp'_s}{(1+e)p'_s} \]

\[ d\varepsilon_d = d\varepsilon^e_d + \frac{2\eta}{|M^2 - \eta^2| + \sigma\eta^2} \left[ \lambda^* - \kappa^* \right] + b \left[ \langle \Delta e - c \rangle + \frac{\gamma\eta\Delta e}{M^* - \eta} \right] \frac{dp'_s}{(1+e)p'_s} \]

**Incremental stress and strain relationship:**

1. Simple and explicit: \( d\varepsilon = f(dp', dq, dp's) \);
2. D matrix essential the same as MCC model and available for direct FEM implementation.

**Ready for application**
If the soil has no structure, then $\Delta e = 0$, $p'_s = p'_o$. 

\[ d\varepsilon_v = d\varepsilon_v^e + (\lambda^* - \kappa^*) \frac{dp'_s}{(1+e)p'_s} + \left[ \langle \Delta e - c \rangle + \frac{\gamma \eta \Delta e}{M^* - \eta} \right] \frac{b dp'_s}{(1+e)p'_s} \]

\[ d\varepsilon_d = d\varepsilon_d^e + \frac{2\eta}{|M^* - \eta|^2 + \alpha \eta \left(1 - \sqrt{\frac{p'_o}{p'_s}}\right)} \times \left\{ (\lambda^* - \kappa^*) + b \left( \frac{\gamma \eta \Delta e}{M^* - \eta} \right) \right\} \frac{dp'_s}{(1+e)p'_s} \]
If the soil has no structure, then $\Delta e = 0$, $p'_s = p'_o$.

**Modified Cam Clay Model**

\[
d\varepsilon_v = d\varepsilon^e_v + \left(\lambda^* - \kappa^*\right) \frac{dp'_s}{(1+e)p'_s} + \left[\Delta e - c\right] + \frac{\gamma\eta\Delta e}{M^* - \eta} \frac{bdp'_s}{(1+e)p'_s}
\]

\[
d\varepsilon_d = d\varepsilon^e_d + \frac{2\eta}{\left|M^* - \eta^2\right| + \sigma\eta^2} \left[\left(\lambda^* - \kappa^*\right) + b\left(\Delta e - c\right) + \frac{\gamma\eta\Delta e}{M^* - \eta}\right] \frac{dp'_s}{(1+e)p'_s}
\]
4.6 Incremental stress and strain relationship

If the soil has no structure or the influence of soil structure is negligible, the SCC model reduced to the MCC model.

SCC model = MCC model in every respect
4.7: Model parameters

10 parameters: $M^*$, $\Gamma^*$, $\lambda^*$, $\kappa^*$, $\nu^*$ and $b$, $c$, $\gamma$, $\varpi$, $\rho'_{y,i}$.

5 old parameters: describing the intrinsic soil properties, the same as MCC model and well studied. Not discussed.

5 new parameters: $b$, $c$, $\gamma$, $\varpi$, $\rho'_{y,i}$, describing the effect of soil structures.
4.7: Model parameters

10 parameters: $M^*$, $\Gamma^*$, $\lambda^*$, $\kappa^*$, $\nu^*$ and $b$, $c$, $\gamma$, $\varpi$, $p'_{y,i}$.

$b$, $c$, $p'_{y,i}$: determined from an isotropic compression test or estimated from an odometer test.

$p'_{y,i}$: initial yield stress;

$b$: destructuring index;

$c$: the part of $\Delta e$ that cannot be eliminated by the increase of stress level.
4.7: Model parameters

10 parameters: $M^*$, $\Gamma^*$, $\lambda^*$, $\kappa^*$, $\nu^*$ and $b$, $c$, $\gamma$, $\varpi$, $p'_{y,i}$.

$\gamma$: describing the reduction in additional voids ratio in relation to the current level of shear stress, determined by plotting test data in the $e - \ln p'$ space.

$\varpi$: the effect of additional voids ratio on flow rule.

Model parameters: all determined from conventional tests conveniently.
5: Validation of the model

The SCC model used to simulate the behaviour of structured soils

Soil types: clays, sands, calcareous soils and clayshale.

Tests: isotropic compression, drained & undrained shearing tests.

Evaluation of the SCC model
5.1 Drained behaviour of a natural calcarenite

Test data: Lagioia & Nova (1995)
Natural calcarenite: uniform marine deposit and with very sensitive structure.
Samples: intact and reconstituted.
Triaxial compression tests with $\sigma'_3 = \text{constant}$,
$\sigma'_3 = 50 \text{ kPa} ---- 5000 \text{ kPa}$. 
Determination of model parameters

Parameters

$\lambda^* = 0.208$
$E^* = 77000$ kPa
$e^*_{IC} = 2.38$
$b = 30$
$c = 0$
$p'_{y,i} = 2,400$ kPa

Others:
$M^* = 1.45$
$\nu^* = 0.13$
$\omega = 4$
$\gamma = 2.1$

Isotropic compression test
Soil, highly over-consolidated, OCR=100, Stiff, basically elastic, Reach peak strength, Softening to failure

Pattern: similar
Cyclic test, Before yielding & unloading and reloading: high stiffness & recoverable

$\sigma'_3 = 600$ kPa

$\varepsilon_d$ = Deviatoric strain

$\sigma'_3$ = Deviatoric stress

$\varepsilon_v$ = Volumetric strain

$\sigma'_3$ = 600 kPa
Before yielding:
elastic;

A very special feature:
at yielding, a large amount of plastic defm
(ε_v, ε_d) produced at constant stress

Successfully captured by this simple model
A large amount of plastic deformation accumulated at constant stress.

By the model, for the soil, $b=30$ is highly sensitive.

When yielding occurs, destructuring at almost no change of stress.
Structure of the soil
\[ \rho'_s = 2400 \text{ kPa}, \ b = 30, \]
For \( \sigma'_3 = 3500 \text{ kPa}, \) Structure removed
Soil reconstituted
Model gives good prediction of both structured and reconstituted calcareous soil.
5.2 Drained behaviour of a clayshale

Data from Wong (1980): Natural stiff clayshale,

Drained triaxial compression tests with $\sigma'_3 =$ constant,
$\sigma'_3 =$ 50 kPa ---- 500 kPa.
5.2 Drained behaviour of a clayshale

Data from Wong (1980): Natural stiff clayshale,

Model Parameters

\[ M^* = 1.45, \quad \lambda^* = 0.06, \quad E^* = 73,000 \text{ kPa}, \quad e_{lc}^* = 0.668, \]
\[ \nu^* = 0.25, \]
\[ b = 0.2, \quad c = 0, \quad \omega = 1, \quad \gamma = 1, \quad p'_{y,i} = 3,700 \text{ kPa}. \]
Over-consolidated soil behaviour: Peak strength & then softening.

Before reaching peak strength, i.e., inside yield surface, elastic, the behaviour of the three tests the same \( \Rightarrow \) constant Young’s modulus \( E \).
A special feature, observed in tests on most natural soft clays, Test $\sigma'_3 = 50$ kPa
During softening, compressive volumetric deformation seen

This typical feature for soft clay captured by the SCC model.
In Cam Clay model, MCC model and many models, volumetric deformation ALWAYS EXPANSIVE for softening.
5.3 Undrained behaviour of a natural soft clay

Test data: Lacasse et al (1985)

Emmerstad Clay: Natural soft clay
High sensitivity, $S_t$ from 60 to infinitive.

Undrained tests both compression tests and extension tests

soil sheared from initial anisotropic stress state: (the initial stress state = in situ stress).
5.3 Undrained behaviour of a natural soft clay

Test data: Lacasse et al (1985)

Emmerstad Clay: Natural soft clay

Model Parameters

\[ \begin{align*}
\lambda^* &= 1.37, \\
\kappa^* &= 0.07, \\
e_{ic}^* &= 0.006, \\
\nu^* &= 0.25, \\
b &= 0.4, \\
c &= 0, \\
\omega &= 1.8, \\
\gamma &= 0.1, \\
p_{y,i}' &= 98 \text{ kPa.}
\end{align*} \]

For soft clays, \( c = 0 \).
Stress paths for compression and extension tests

Highly sensitive soft clay,
Model predicts final failure at near zero shear strength, confirmed by tests
Overall, the model describes very well the unusual behaviour of this particular clay.
For undrained triaxial compression test:
(1) Stress path initial raises up as elastic behaviour, A1;
(2) Yielding and softening occurred at 1. Volumetric deformation expansive $\Rightarrow$ negative pore pressure, stress path bends toward the right side and above CSL.
For undrained triaxial compression test:

(3) At 2, stress path changes direction and travels above CSL and towards the zero stress state. Positive pore pressure produced, indicated compressive volumetric deformation.

(4) Finally, the soil finally fails at zero shear strength.

Successfully modelled by SCC model.
For undrained triaxial extension test:
Similarly behaviour observed
A1: inside the structural yield surface, basically elastic;
12: negative pore pressure $\Rightarrow$ volumetric expansion;
23: positive pore pressure $\Rightarrow$ volumetric compression;
Failure at near zero shear strength
Data from Golightly and Hyde (1988)
Dogs Bay Carbonate sand, prepared by dry-pluviating method.
Undrained triaxial compression tests with \( \sigma_3 = \) constant
\( \sigma'_3 = 150 \text{ kPa} ---- 900 \text{ kPa}. \)
5.4 Undrained behaviour of a carbonate sand

Dogs Bay Carbonate sand

Model Parameters

Parameters for material intrinsic properties

\[ M^* = 1.75, \; \lambda^* = 0.135, \; E^* = 100,000 \text{ kPa}, \]
\[ e^*_{lc} = 1.87, \; \nu^* = 0.25, \]

Parameters for material intrinsic properties

\[ b = 0.25, \; c = 0, \; \omega = 0.1, \; \gamma = 0.2, \; \rho'_{y,i} = 400 \text{ kPa}. \]
4 tests: \( \sigma'_{3} = 150, 350, 600, 1000 \text{ kPa}. \)

(1) Effective stress path;

(2) Pore pressure and axial strain

The model captures the main features well.
Special features of carbonate sand behaviour, not seen in natural clays

For soil on the wet side, stress state on the yield surface,
(1) Virgin yielding start, stress path travels along yield surface upwards and meets critical state strength;
(2) Or initial elastic behaviour reaches yield surface, then follows surface.
Special features of carbonate sand behaviour, not seen in natural clays

For soil on the wet side, stress state on the yield surface,
(3) At CS strength, stress path changes direction and travels along the critical state line and upwards.
(4) All tests, no matter initial virgin yielding or softening behaviour, stress paths upwards and along CSL when reaching critical state strength.
6: Performance for boundary value problems

Implemented into the finite element programs such as AFENA ET AL

Solving practical geotechnical problems:
- bearing capacity of foundations on natural soils,
- settlement of embankment on natural soils,
- response of natural soils to cone penetration
6: Performance for boundary value problems

Compared with field test data, the results satisfactory

One example:

The influence of soil structure on the settlement and bearing capacity of rigid footing is shown here.
Load-displacement response of rigid footing after (K. Islam, 2004)
Because natural clay more compressible than reconstituted clay, 5 to 20 times for soft clays. MCC gives much higher stiffness and bearing capacity than actual soil.

→ Serious over-prediction, unsafe
7: Summary

The behaviour of soil found in nature differs remarkably from that of the same soil in laboratory reconstituted states.

When solving practical geotechnical engineering problems, the constitutive models developed for describing the behaviour of laboratory reconstituted soil are not good enough.

Influence of soil structure must be considered
7: Summary

A simple predictive model
the Structured Cam Clay (SCC) model
Aim of the model: to provide tool for the solution of boundary value problems encountered in geotechnical engineering.
Simple and convenient for engineers
7: Summary

We select Modified Cam Clay (MCC) model as the base for the new model.

(1) simple and rational, yet describes the behaviour reconstituted soil with acceptable accuracy;

(2) widely applied in geo-engineering field.
7: Summary

defined in a 4-D space: e, current voids ratio, 
(p′, q), current stress state, 
stress history, and 
soil structure.
7: Summary

Soil behaviour in the $p'$ - $q$ space divided into two regions by the yield surface.

Elastic & plastic regions

Soil behaves purely elastically for any stress excursion inside the yield surface;
virgin yield occurs for stress state on the surface and causing it expansion.

During virgin yielding, yield surface expands with the current stress stays on the surface.
7: Summary

The compression behaviour of structured soils is described as

\[ e = e^* + \Delta e \]

\( e^* \): voids ratio for the same soil in a reconstituted state

\( \Delta e \): additional voids ratio sustained by soil structure
7: Summary

The compression behaviour of structured soils is described as

Based on experimental data, Liu and Carter (1999, 2000) proposed the following equation for $\Delta e$

$$\Delta e = a \left( \frac{p_{y,i}'}{p'} \right)^b + c$$

$b$: destructuring index;
$c$: the part of $\Delta e$ that cannot be eliminated by the increase of stress level;
$a$: $\Delta e_i = a + c$. 
7: Summary

Some basic assumptions are:

(1) The mechanical properties of a clay in laboratory reconstituted states are treated as intrinsic, which can be described adequately by the MCC model.

(2) Elastic properties of soil are independent of soil structure.

(3) Both hardening and destructurig of soil are dependent on plastic volumetric deformation.

With the proposed isotropic compression line, the SCC model is formulated.
7: Summary

The Structured Cam Clay (SCC) model applied to simulate the behaviour of soil for (1) laboratory single element tests and (2) boundary value problems.

(1) Convenient identification of model parameters and for implementation into numerical analysis

(2) Successfully captures many important features of the behaviour of structured soils and influence of soil structure
7: Summary

(3) Significantly improves the performance of the Modified Cam Clay Model, represents well the behaviour of real soil, the soil found in nature.

(4) Useful tool for the solution of boundary value problems encountered in geotechnical practice.
Thank you very much!