Soil Plasticity and the Structured Cam Clay Model

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Soil Plasticity and the Structured Cam Clay Model

Summary

I: Soil plasticity

II: The Structured Cam Clay model

III: Further development within the theoretical framework of the SCC model

Soil Plasticity and the Structured Cam Clay Model

I: Soil plasticity

1: Knowledge and wisdom from past achievements
2: Constitutive models and its importance
3: General Elasticity
4: Original Cam Clay model and soil plasticity
5: Models of the Cam Clay family
6: Questions and discussions at any time welcome!

II: The Structured Cam Clay model
III: Further development within the theoretical framework of the SCC model

Soil Plasticity

1: Knowledge and wisdom from past achievements

The earth, Mother Earth, made up of rocks and soils

➔ the research of geotechnical engineering

The great soil for human being:

Homer: Greek poet,

Saddest of life: death

He cannot see the soil of his homeland again.

He can never smell the smells of home soil.

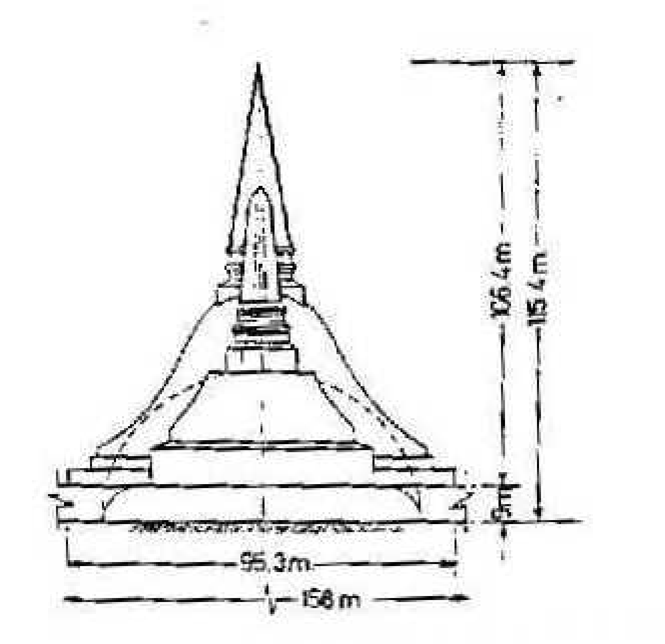
He can touch the soil any more.

The earth supports human being physically and spiritually. Our ancestors knew soil well.

Our ancestors knew soil well.
Three examples to see the knowledge and wisdom:
A pagoda, a wall, and an merchant

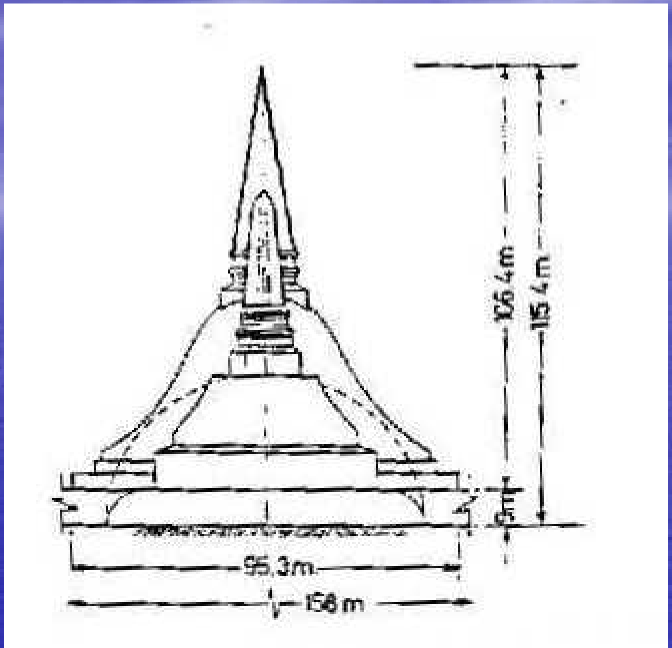
The pagoda: the Pagoda of Phra Pathom Chedi, Thailand
The Wall: the Great Wall of China
A merchant: a British merchant

Pagoda of Phra Pathom Chedi, Thailand



Built in 300 BC;
Height: 115 m
Diameter of the circular base: 158 m
Weight: 500,000 ton

Pagoda of Phra Pathom Chedi, Thailand

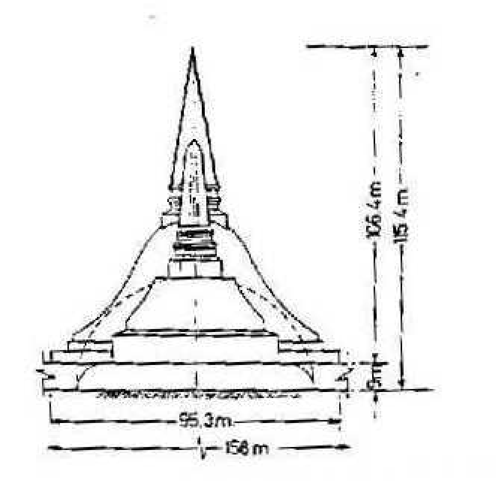


Built in 300 BC;
Size: 115 m × 158 m × 158 m;
Weight: 0.5 MT

After 2300 years
Stand well with a uniform settlement of 2.5 m

Engineering miracle.
The load concentration of the pagoda: 10 MN/m^2
European design code: 4-5 MN/M^2 for high building;
1.5 – 4 MN/M2 for large silos.

Pagoda of Phra Pathom Chedi



Engineering miracle.

Knowledge: (1) the non-uniform settlement, not the uniform settlement, that destroys our structure. (2) The bearing capacity of the ground increases with settlement.

Modern numerical analysis gives the same conclusion. But modern engineers dare not design such a building.

The Great Wall of China

Built around 250 BC as military defense project

The Great Wall of China

Techniques used for construction

(1) Wood hammer to density the soil

They knew:

Strength of soil

increase with density

Stiffness of deformation

A very important research topic in 1960s.

The Great Wall of China

Techniques used for construction

(2) Reinforcement by grass and tree

They knew how to improve the strength of soil and stiffness of deformation by geo-textile reinforcement

The Great Wall of China

Techniques used for construction

(3) Cementation by sticky rice

cementing gravel and rocks to give strong support

The Great Wall of China

Reinforcement by strong materials and by cementation: very popular methods of soil improvement in our modern world since 1970.

Only 1980, research in improving soil strength and stiffness by reinforcement and cementation The work in Thailand is world class (e.g., AIT, here by Dr. Horpibulsuk)

A British corn merchant
Darwin (1883) and Renald (1887) made an important discovery of soil property

When sheared, loose soil will reduce in volume, but a dense soil will increase in volume.

Characteristic of soils, called dilatancy, fundamental different from other materials. A great challenge to geotechnical engineers.

Rowe (1962) performed a "pioneering" study on dilatancy and proposed a dilatancy law, a milestone in modern soil mechanics.

A British corn merchant
Darwin (1883) and Renald (1887) noticed.
"Corn merchants have known dilatancy a very long time ago".

Corns bought and sold by volume.

A common practice:
If you are a buyer, you will shake it.
If corns decrease in volume, it is loose. You shake it to make it small.
If the volume increases, it is dense.   You leave it alone, you do not touch it.

A good business sense.

Conclusion

(a) Variation of the peak strength of soil with density and stress level, (b) influence on the stiffness of soil to resist deformation, and (c) the dilatancy of soil.

Three major challenges, investigated in tests, modelling by theory.

(1) Our knowledge of soil mechanics comes from human practice.

(2) Our research on soil mechanics is to help human practice. Only by serving human need, soil mechanics is a live theory.

2: Constitutive models and its importance

Constitutive model:

describing the change in the strain state of an element of material to the change in the stress state acting on the element.

$$d\varepsilon = f\left(\xi, \chi, \gamma, \sigma', d\sigma'\right)$$

Strain increment

Stress increment

Current stress state

Material properties

Strain history

Stress history

Constitutive model:

$$d\varepsilon = f\left(\xi, \chi, \gamma, d\sigma'\right)$$

Providing information on

(strength + deformation) of a material in an infinitesimal element

Only through the knowledge of the strength and deformation of an element of material → strength and deformation of an engineering structure

No other way!!

Constitutive model:

The strength and deformation of a structure.

That is all of An engineer’s work, no more.

This is the important and the reason for constitutive modelling

3: General Elasticity

Terzaghi (1936):

Effective stress principle

$$\sigma' = \sigma - u$$

Pore pressure

Effective stress

Total stress

It is the effective stress that determines the deformation of soil!

3: General Elasticity

Terzaghi (1936): Effective stress principle

$$\sigma' = \sigma - u$$

The principle of effective stress: foundation of modern soil mechanics;

Terzaghi: founder of modern soil mechanics.

Because it enables us make a sensible link between the deformation of soil and the stress acting on it.

Again for constitutive modelling

3: General Elasticity

With the effective stress principle, theory of elasticity introduced into soil mechanics.

Elastic deformation proposed by Hook (1678)

"Deformation is proportional to force".

At the element level

Effective stress applied to soil

$$d\varepsilon = \frac{d\sigma'}{E}$$

Strain increment

Stiffness, a material constant

Original Hook’s law

Elastic response of soil to loading dependent on mechanical properties of the soil.
material constant, Young’s modulus.

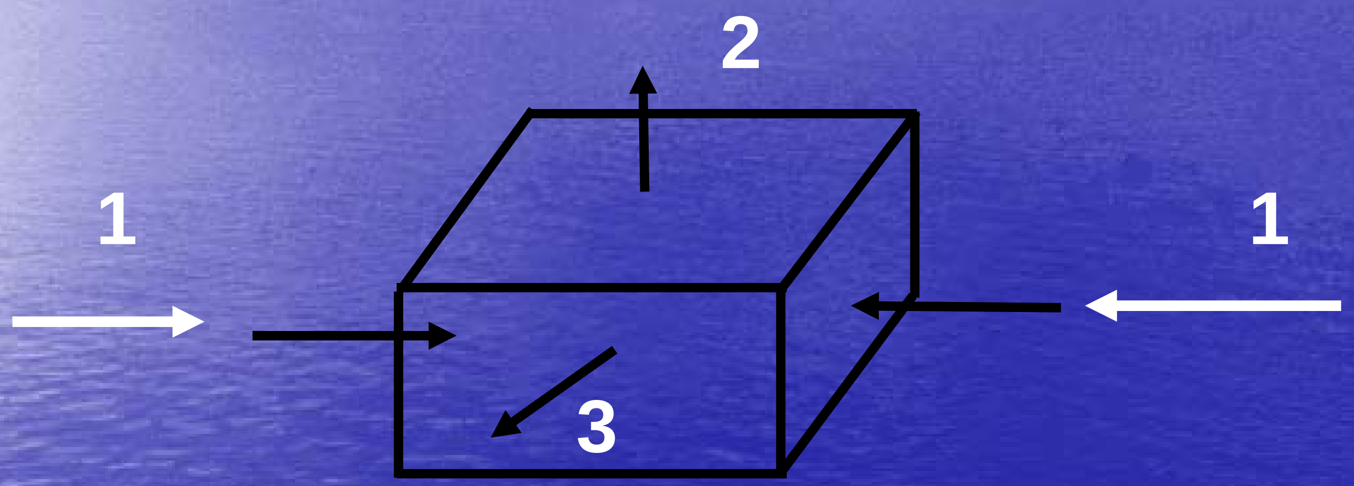
$$d\varepsilon = \frac{d\sigma'}{E}$$

Stiffness, a material constant

Original Hook's law:

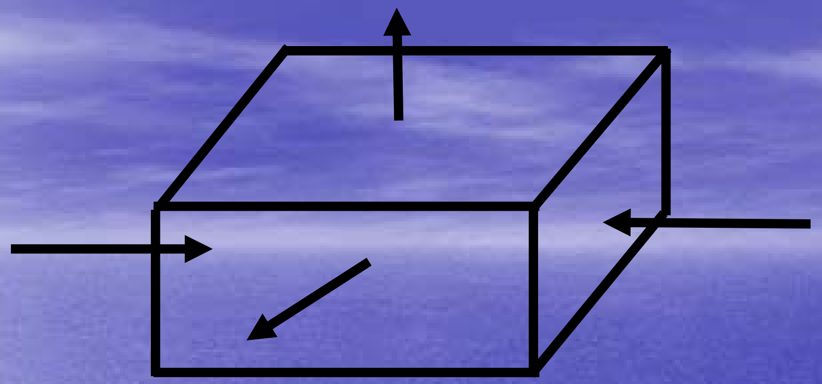
$$d\varepsilon = \frac{d\sigma'}{E}$$

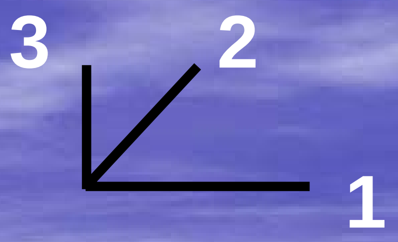
Experimental observation: deformation also found at direction vertical to the applied stress



Applying stress in direction ➔ strain in directions 1, 2, and 3

Lateral deformation added





Coordinate

Hence for stress σ'_1,

$$d\varepsilon_1 = \frac{d\sigma'_1}{E_1}$$

Young's modulus E in direction 1

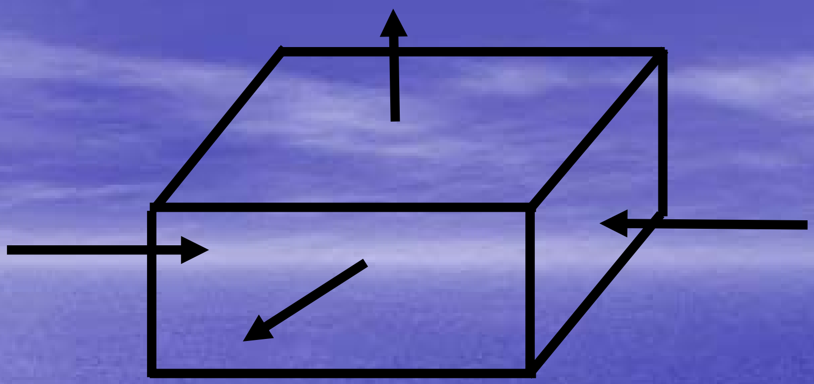
$$d\varepsilon_2 = \nu_{12}\frac{d\sigma'_1}{E_1}$$

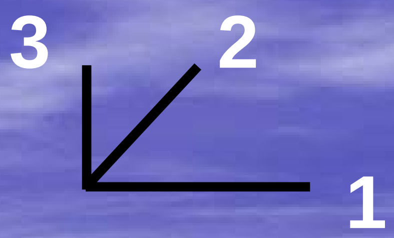
Poisson's ratio ν for deformation in direction 2 by stressing in direction 1

$$d\varepsilon_3 = \nu_{13}\frac{d\sigma'_1}{E_1}$$

Poisson's ratio ν_{13}

Elastic deformation





Coordinate

Hence for stress σ'_1,

$$d\varepsilon_1 = \frac{d\sigma'_1}{E_1} \qquad d\varepsilon_2 = \nu_{12}\frac{d\sigma'_1}{E_1} \qquad d\varepsilon_3 = \nu_{13}\frac{d\sigma'_1}{E_1}$$

Deformation with three principal stresses σ'_1, σ'_2, and σ'_3,

Elastic, linear and small deformation

➔ deformation superimposed.

Elastic deformation

General elastic equation with σ′$_1$, σ′$_2$, and σ′$_3$

$$d\varepsilon_1 = \frac{d\sigma'_1}{E_1} - \nu_{21}\frac{d\sigma'_2}{E_2} - \nu_{31}\frac{d\sigma'_3}{E_3}$$

$$d\varepsilon_2 = \nu_{12}\frac{d\sigma'_1}{E_1} - \frac{d\sigma'_2}{E_2} - \nu_{32}\frac{d\sigma'_3}{E_3}$$

$$d\varepsilon_3 = \nu_{13}\frac{d\sigma'_1}{E_1} - \nu_{23}\frac{d\sigma'_2}{E_2} - \frac{d\sigma'_3}{E_3}$$

Via principle of virtual work, we know

$\nu_{12} = \nu_{21}$, $\nu_{23} = \nu_{32}$, $\nu_{13} = \nu_{31}$.

General anisotropic elastic equation

$$d\varepsilon_1 = \frac{d\sigma_1'}{E_1} - \nu_{21}\frac{d\sigma_2'}{E_2} - \nu_{31}\frac{d\sigma_3'}{E_3}$$

$$d\varepsilon_2 = \nu_{12}\frac{d\sigma_1'}{E_1} - \frac{d\sigma_2'}{E_2} - \nu_{32}\frac{d\sigma_3'}{E_3}$$

$$d\varepsilon_3 = \nu_{13}\frac{d\sigma_1'}{E_1} - \nu_{23}\frac{d\sigma_2'}{E_2} - \frac{d\sigma_3'}{E_3}$$

$\nu_{12} = \nu_{21}$,
$\nu_{23} = \nu_{32}$,
$\nu_{13} = \nu_{31}$.

General anisotropic elasticity: because

(1) Young's modulus in direction 1, E_1, can be different from E_2 and E_3.

(2) Poisson's ratio for ν_{12}, deformation in direction 2 by loading in direction 1, can be different from ν_{23} and ν_{31}.

General isotropic elastic equation

Isotropy:

Definition: material has no preferred direction.

➔ **E1=E2=E3=E,** $\nu_{12} = \nu_{21} = \nu_{23} = \nu_{32} = \nu_{13} = \nu_{31} = \nu$

Therefore

$$d\varepsilon_1 = \frac{d\sigma_1'}{E} - \nu\frac{d\sigma_2'}{E} - \nu\frac{d\sigma_3'}{E}$$

$$d\varepsilon_2 = \nu\frac{d\sigma_1'}{E} - \frac{d\sigma_2'}{E} - \nu\frac{d\sigma_3'}{E}$$

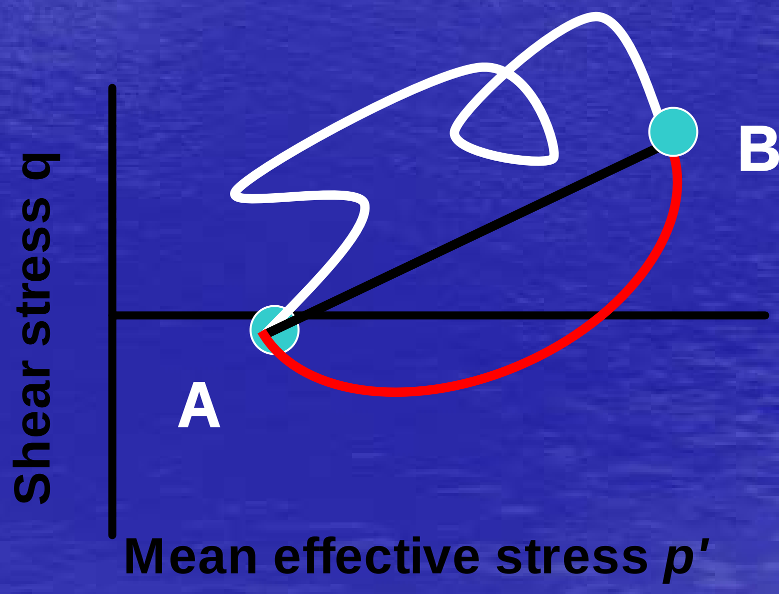
$$d\varepsilon_3 = \nu\frac{d\sigma_1'}{E} - \nu\frac{d\sigma_2'}{E} - \frac{d\sigma_3'}{E}$$

Elastic deformation

Deformation completely recoverable
➔ Elastic deformation dependent on the value of the stress, independent of the stress path, e.g., how soil loaded to the stress

$$\varepsilon_B - \varepsilon_A = \varepsilon\left(E_1, E_2, E_3, \nu_1, \nu_2, \nu_3, \sigma'_B - \sigma'_A\right)$$

Deformation from A to B



4: Original Cam Clay model and soil plasticity

Original Cam Clay model proposed by Roscoe, Schofield and Wroth (1958) revolutional contribution to modern soil mechanics.

Soil mechanics: elegant & consistent science

4: Original Cam Clay model and soil plasticity

Mathematical model:
Object too large, too complicated for us to handle, we idealize it, simplify it into an mathematical model.
Model: represent the features of an object of first importance. Of second importance, ignored.

Model: human invention for understanding and analyzing a reality.
Necessarily simple and idealized.

Original Cam Clay model

formed by study the deformation of soils.

To simplify the work, the deformation of soils in special states: reconstituted clay, not the soil you find in nature.

reconstituted clay: soils taken from nature, dry and breaking to powder, then mixing with water carefully.

Examining experimental result

Simplest one: isotropic compression test

$\sigma'_1 = \sigma'_2 = \sigma'_3$.

Material: reconstituted Kaolin clay

Original Cam Clay model
Isotropic compression test on reconstituted Kaolin

e: **voids ratio**

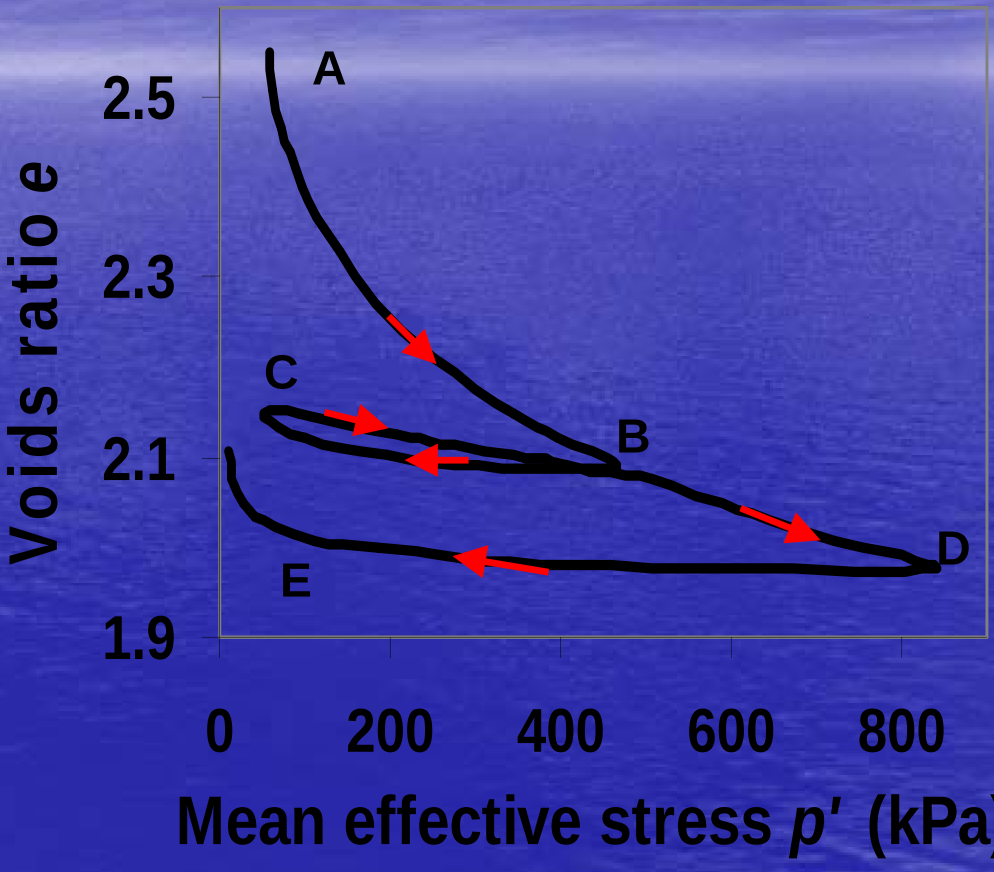
p′: **the mean**

Effective stress

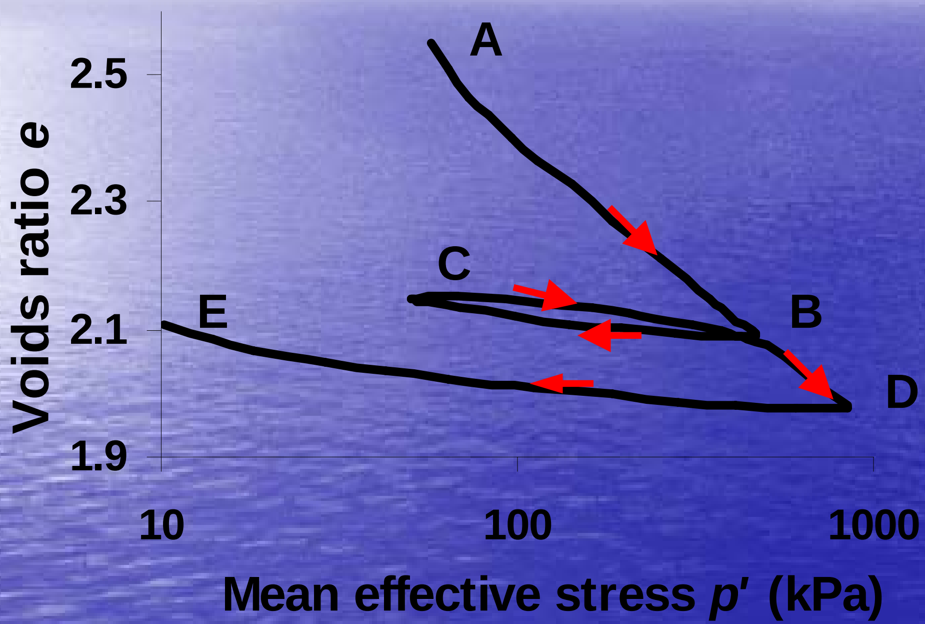
$$p' = \frac{1}{3}\left(\sigma_1' + \sigma_2' + \sigma_3'\right)$$

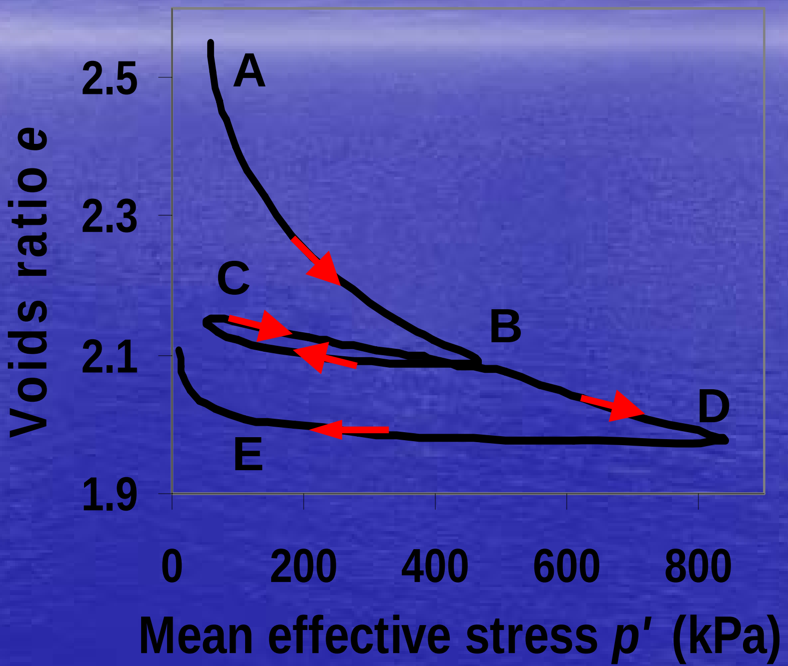
Isotropic test:

$p' = \sigma'_1$.



Isotropic compression test
Two scales: *e*-*p*′ scale and *e*-ln*p*′ scale





The *e* - ln*p*′ scale

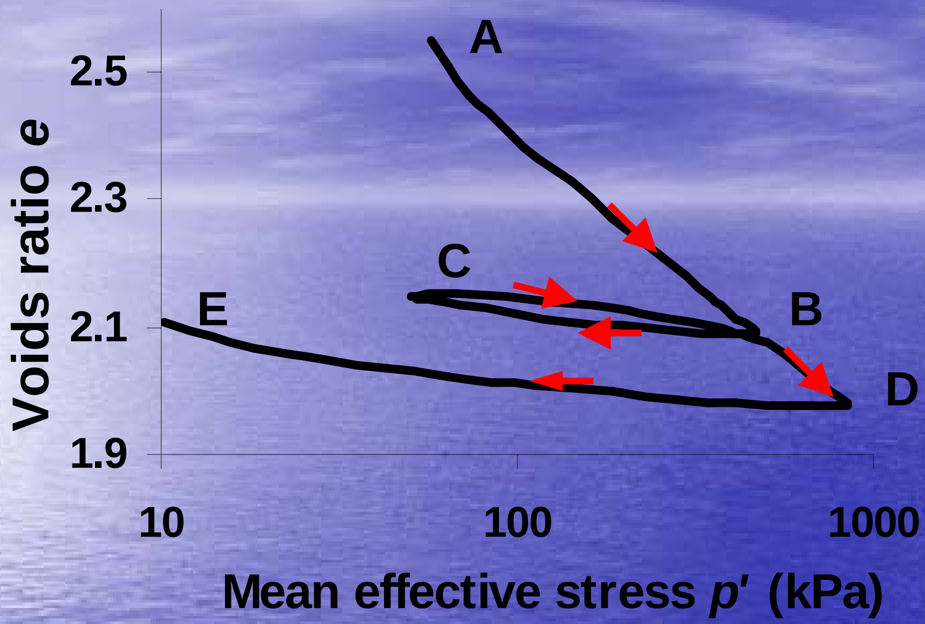
approximately linear

The *e*-*p*′ scale

essentially non-linear

We love linearality, the *e* – ln*p*′ space

Isotropic compression test in the *e*-ln*p′* space



To form a model:

Approximation

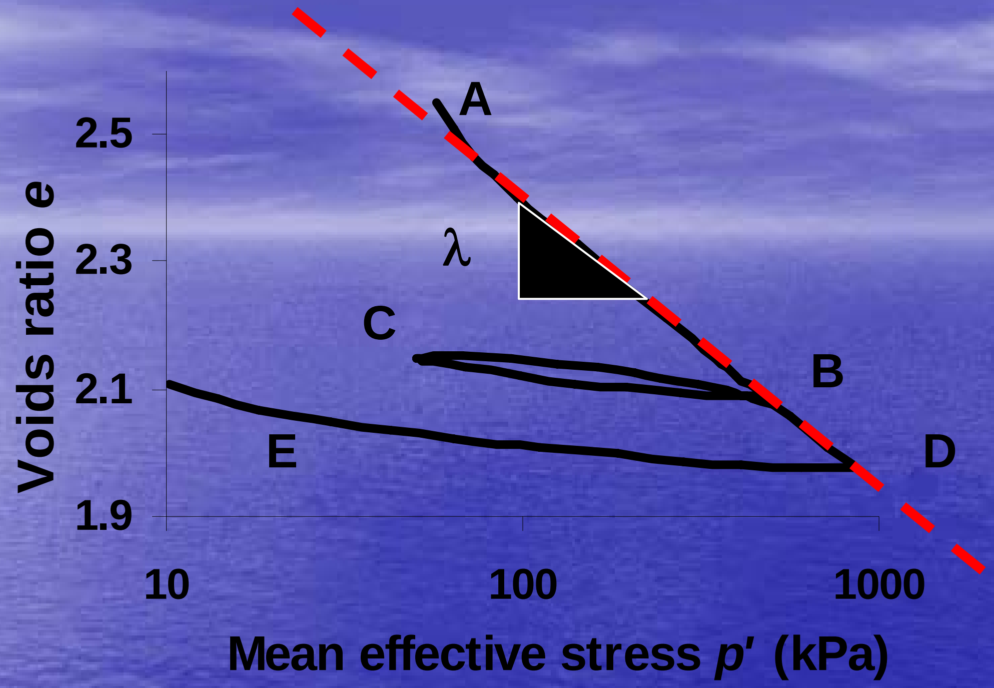
Simplification

Idealization

To represent features of first importance

To form a model for the isotropic compression behaviour of soil

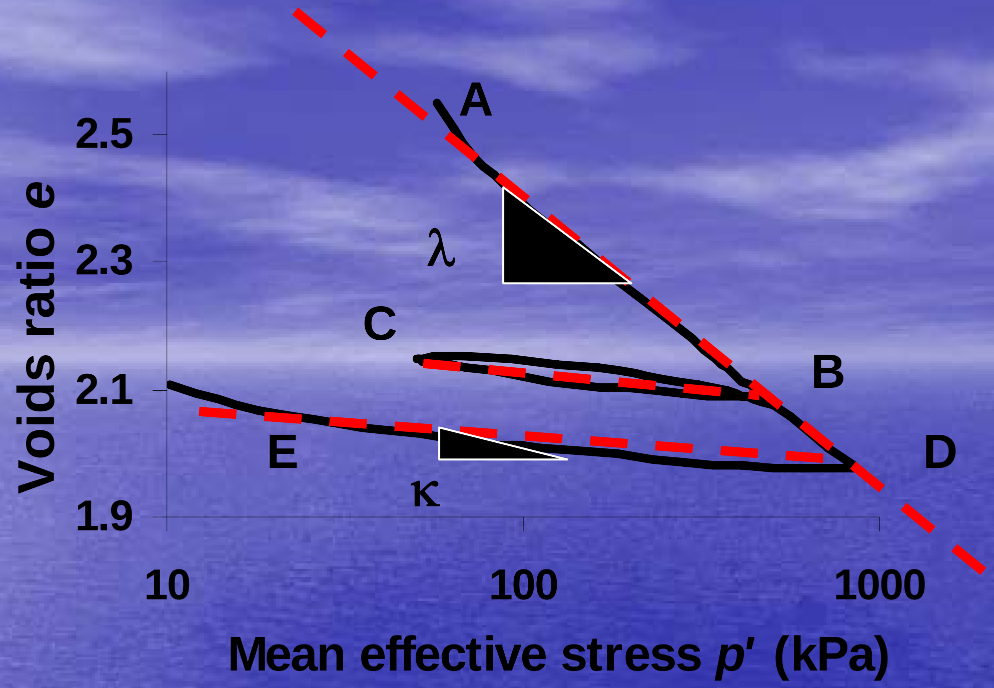
A model for the compression behaviour of soil



Soil behaviour for loading ABD

Idealized as linear in the *e*-ln*p*′ space

Type of loading: current stress is the maximum stress the soil ever experienced ➔ virgin loading

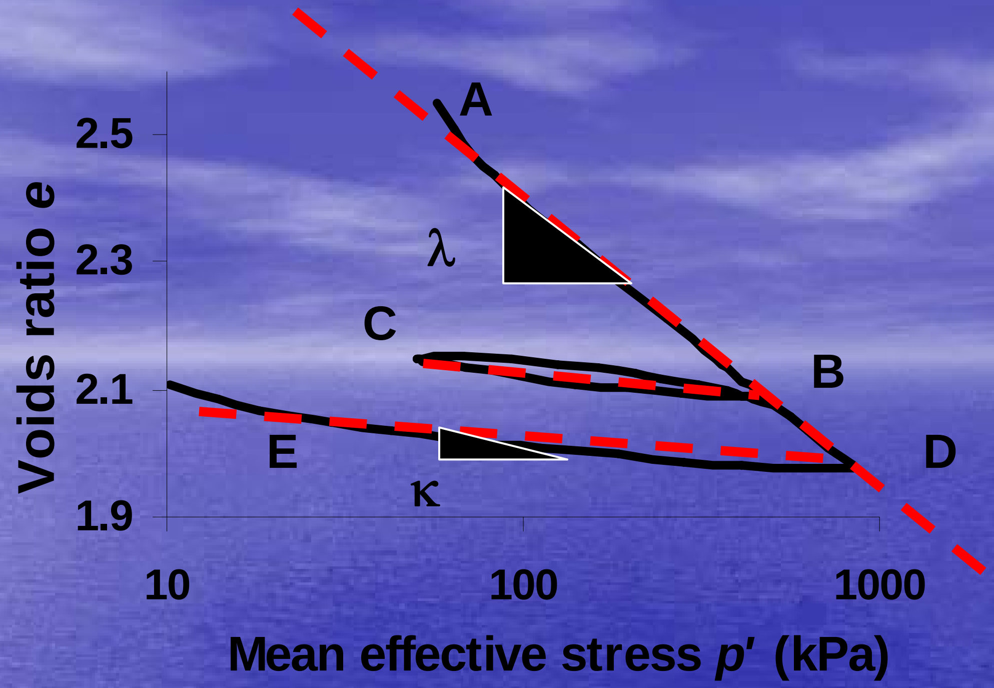


Soil behaviour for unloading BC and DE and reloading CB: also Idealized as linear.

(1) approximately linear for unloading and reloading BC, and unloading DE.

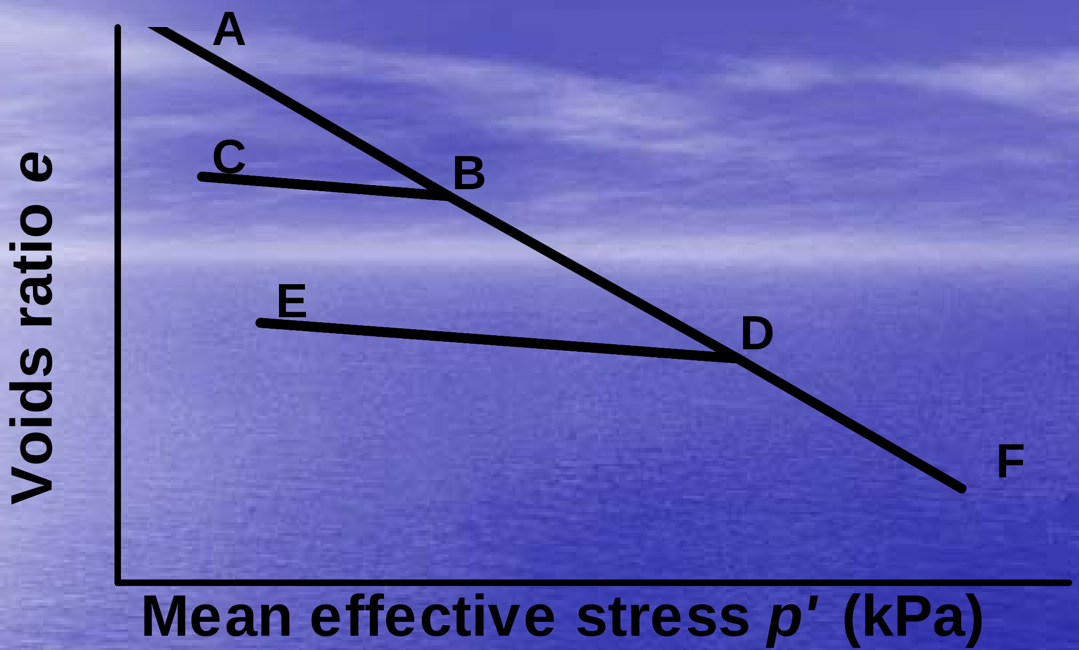
(2) hysteretic loop, of second importance, ignored

(3) The swelling and recompression index are the same.

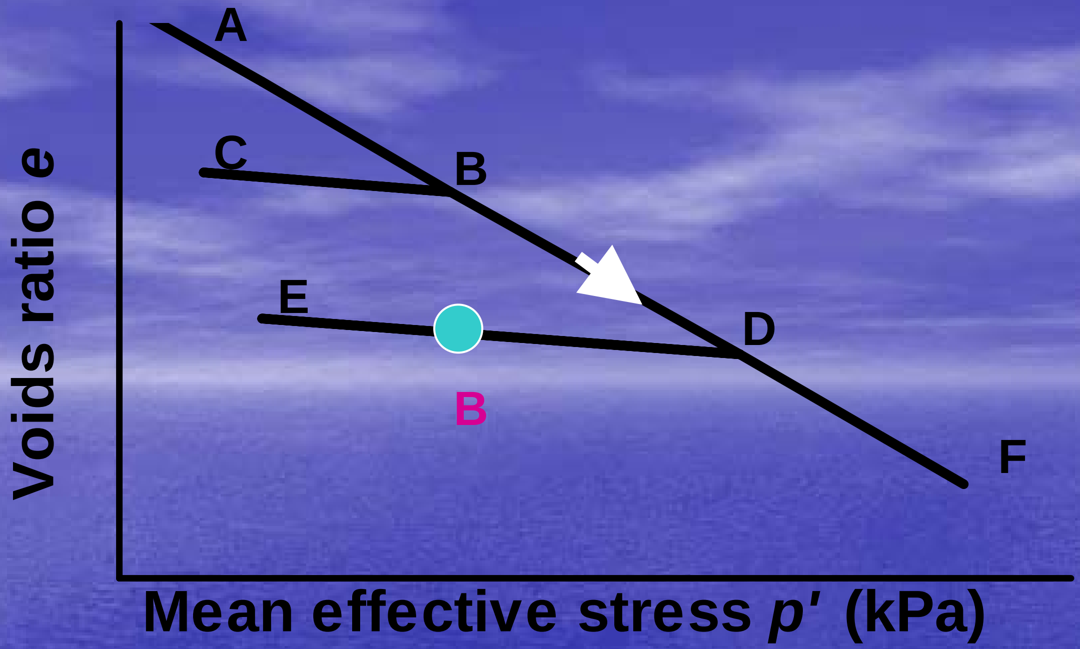


Soil behaviour for unloading BC and DE and reloading CB: also Idealized as linear.

Type of loading: current stress is less than the maximum stress the soil experienced, subsequent loading

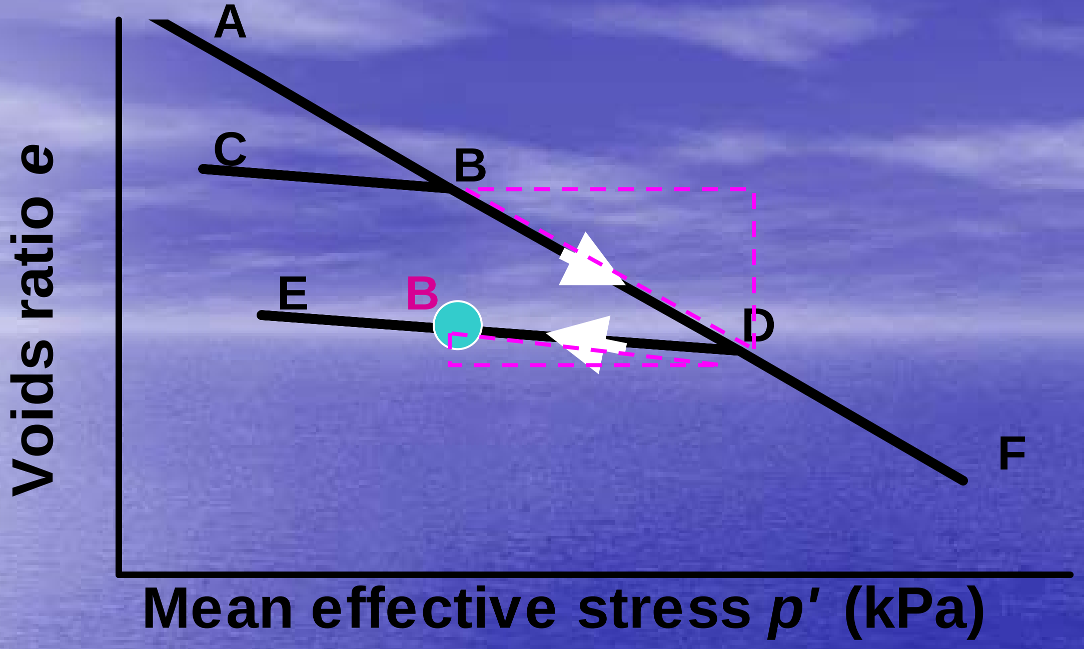


During subsequent loading, soil behaviour for unloading and reloading B➔C➔B, D➔E➔D, deformation completely recoverable
➔ elastic, described by Hook's law



Soil behaviour for reloading C➔B, elastic
When stress goes beyond B, the current stress the maximum stress the soil ever experienced

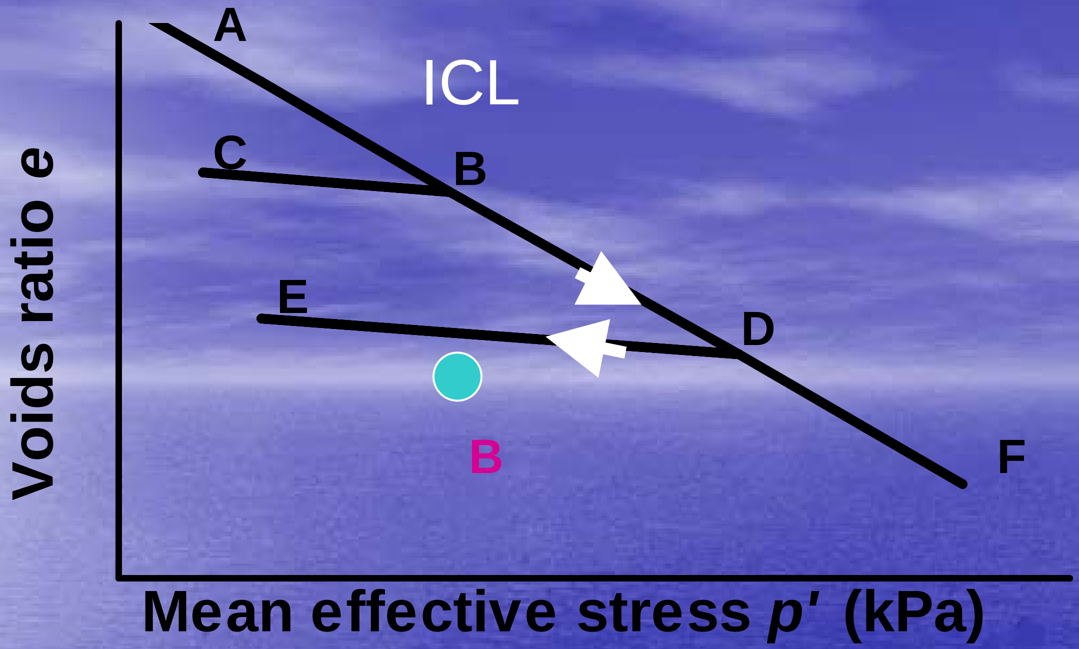
Soil deformation (1) much greater than elastic Deformation;



Loading from B➔D

(2) if loaded from B to D, then unloading from D to B, the stress is less than the maximum stress Elastic deformation, along line DE.

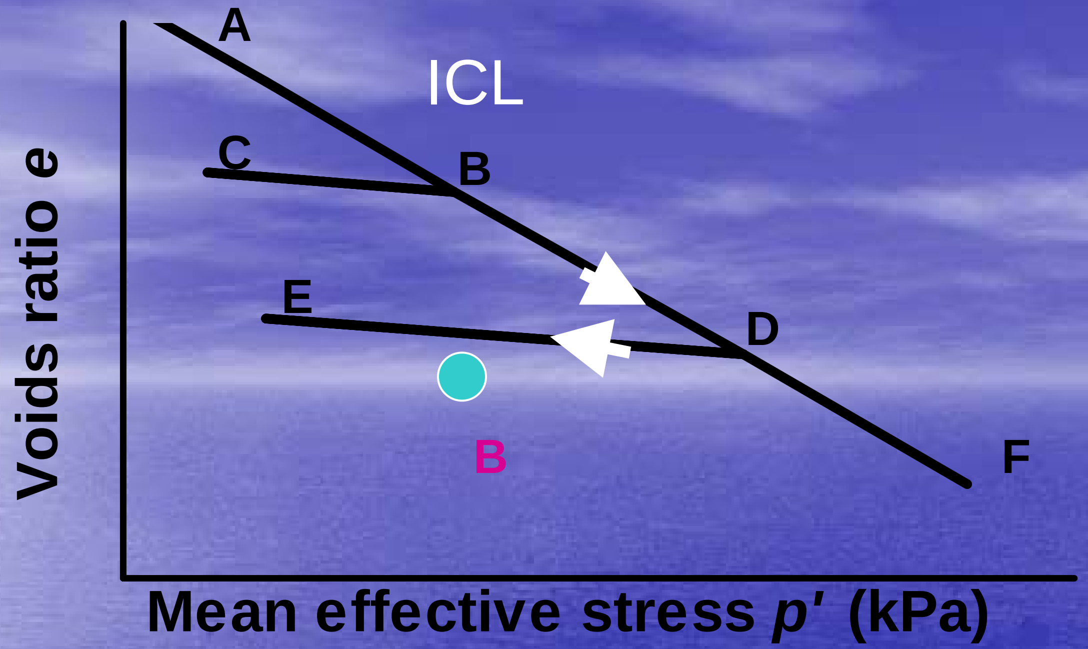
The deformation along first BD loading not completely recoverable, not elastic.



The deformation along first BD loading

not elastic

Both elastic and plastic deformation occurred for loading BD !

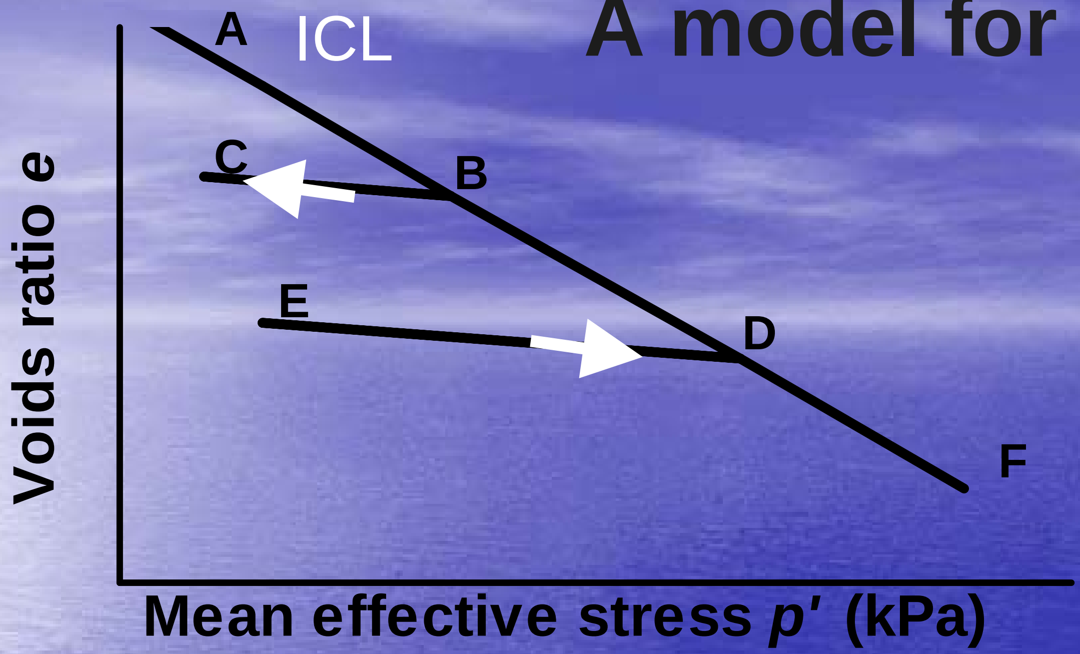


For loading from CBD, at point B, there is a sharp change in the stress and strain curve:

Yielding point

The start point for soil from elastic deformation to plastic deformation

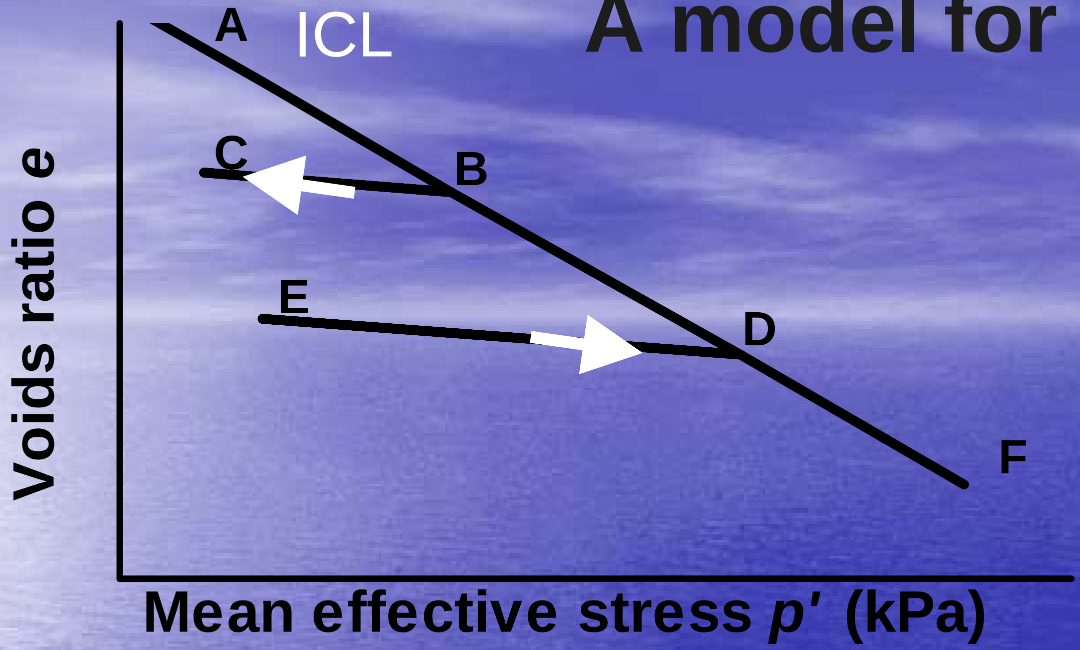
A model for the compression behaviour of soil



For virgin compression, current stress is maximum the soil experienced, soil behaves linearly with gradient λ.

For subsequent loading, current stress less than maximum the soil experienced, e.g., unloading and reloading; soil behaves linearly with gradient κ.

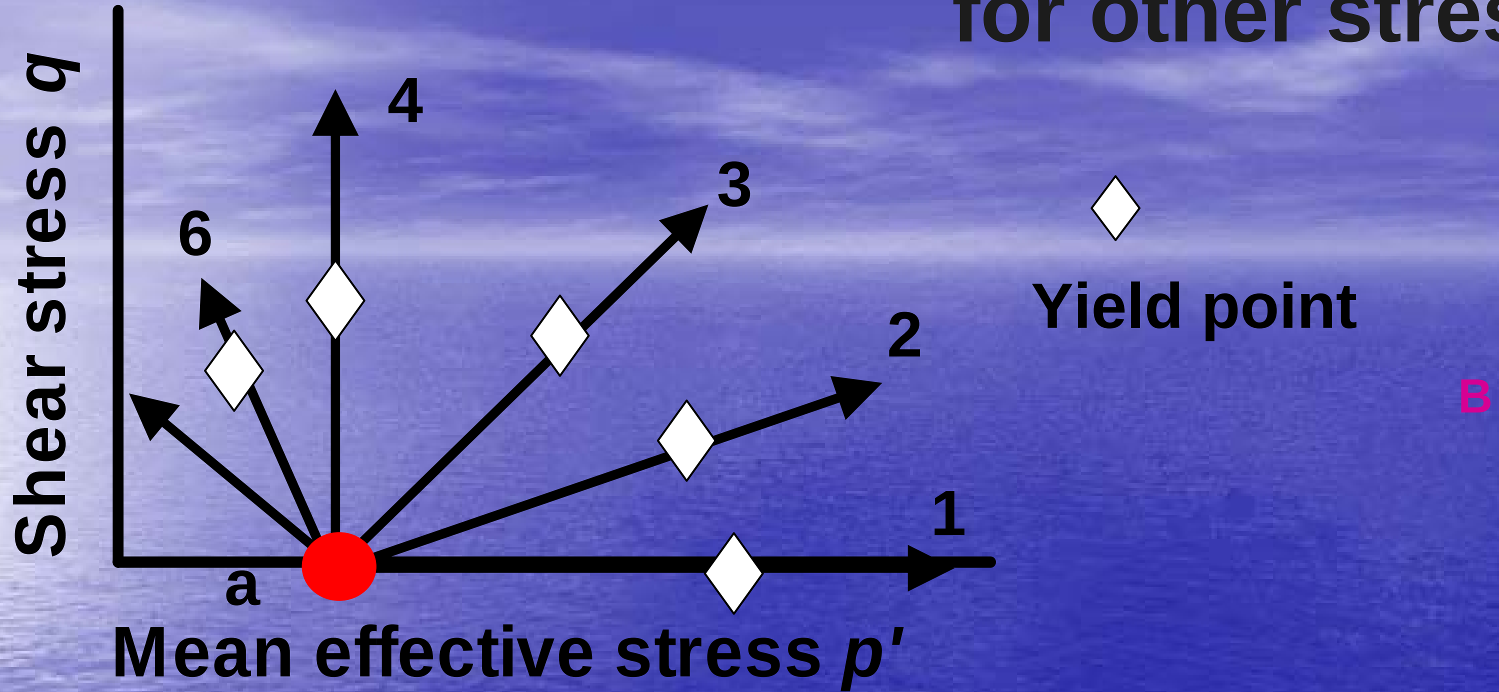
A model for the compression behaviour of soil



For soil behaviour with κ, the deformation is elastic;

For soil behaviour with λ, the deformation is plastic.

Experimental data: soil behaviour for other stress paths



Stress path a1: isotropic compression

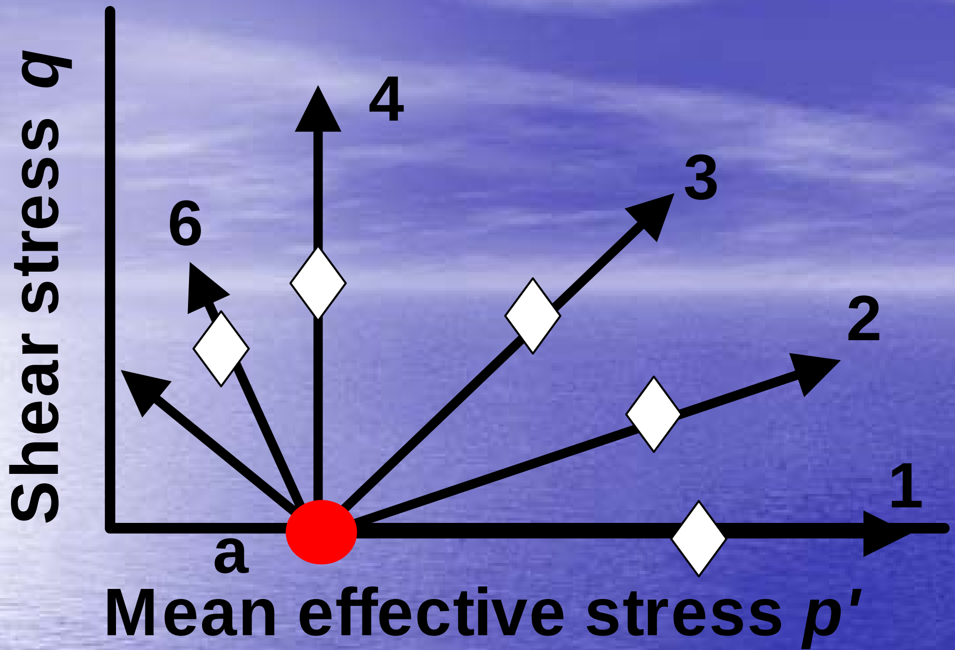
a2, a3, a4: with shear stress

Soil behaviour similar to that for ICL

Soil behaviour into two regions:

pure elastic deformation and plastic deformation

Experimental data: soil behaviour for other stress paths



B

There is a yield point for a given soil for tests along stress paths:

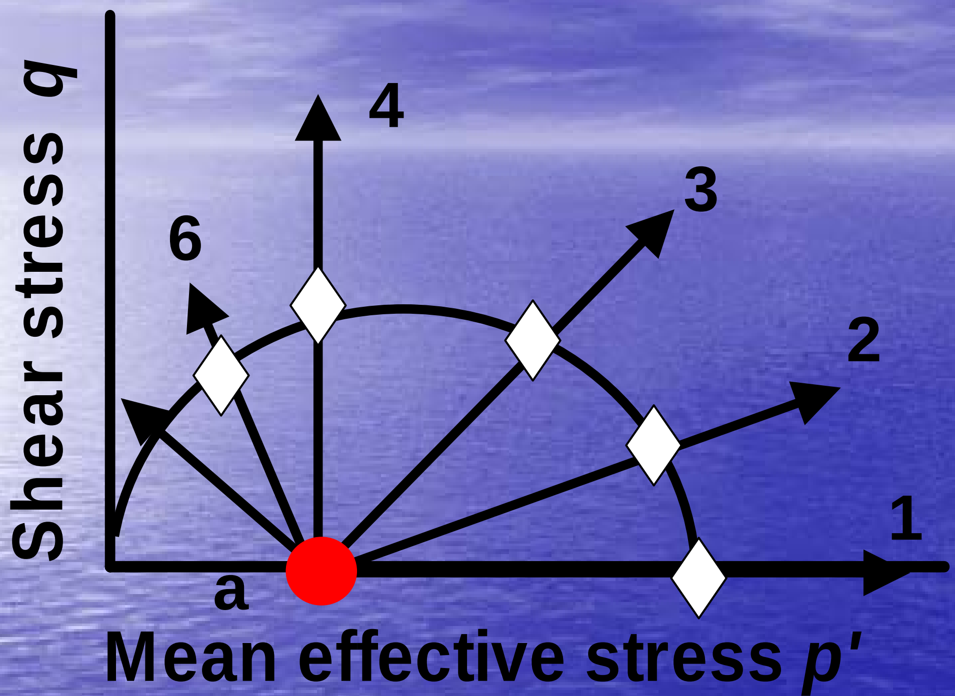
a1, at y1

a2, at y2

a3 , at y3

- - - - - -

Yield surface



All the yielding points make up a boundary in the *p'*-*q* space.

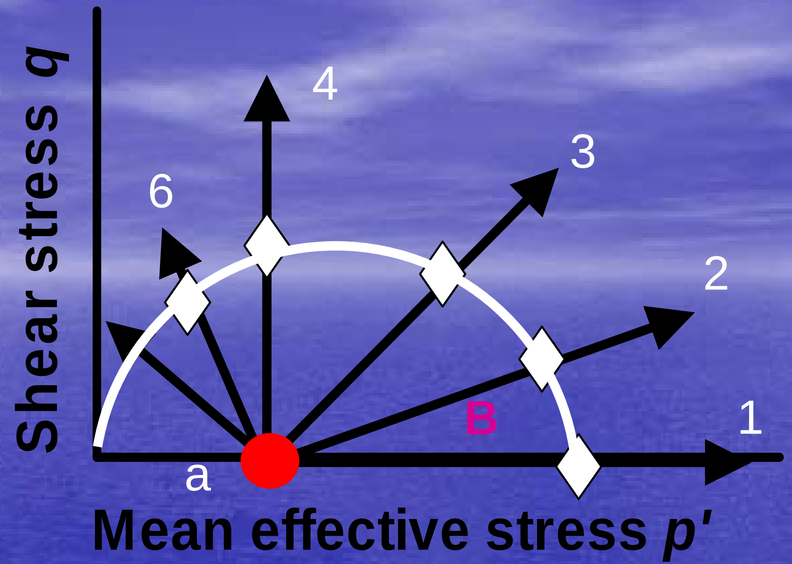
This boundary is the yield surface.

B

(1) Loading inside it pure elastic deformation
(1) Loading outside it plastic deformation.

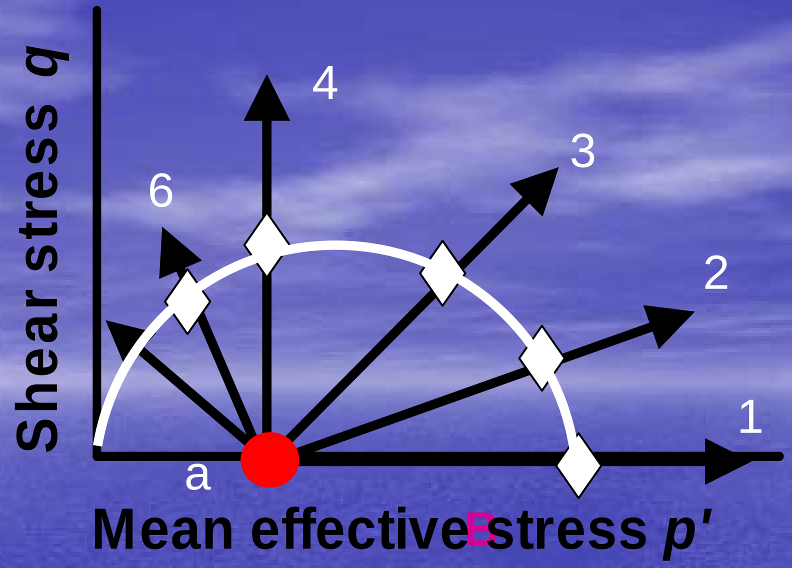
Yield surface

Collection of all the yielding points in stress space



A yield surface divides the behaviour of soil in the stress space into two regions: pure elastic behaviour region and plastic behaviour region

A feature of the yield surface

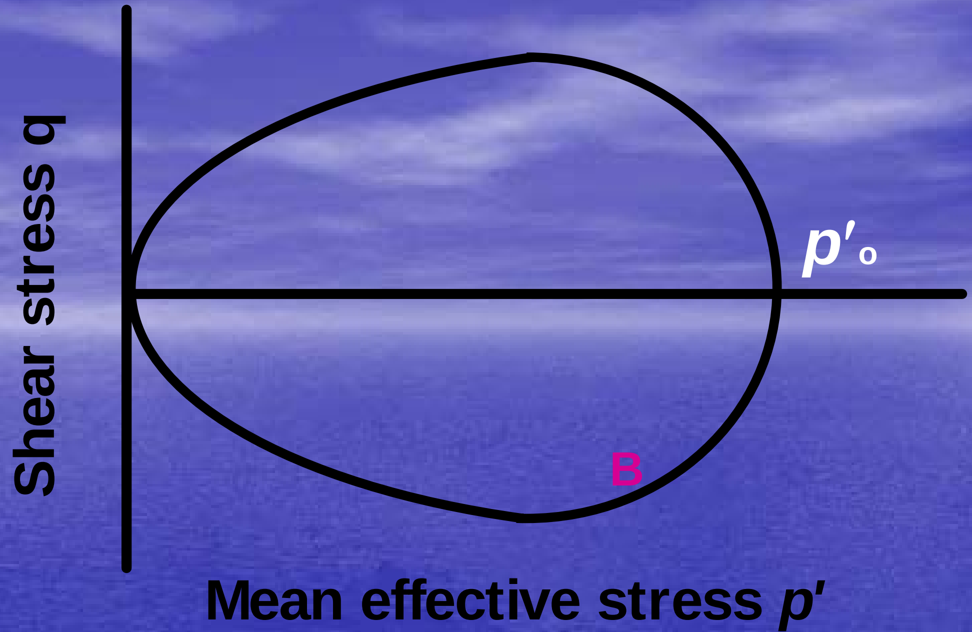


If a soil has no cohesion, e.g., non cohesive soil, tensile force cannot be applied
➔yield surface only for $p' \geq 0$.

Soil a frictional material: stress state (p'=0, q=0) on the yield surface
➔ Yield surface passes through the origin of the stress coordinates.

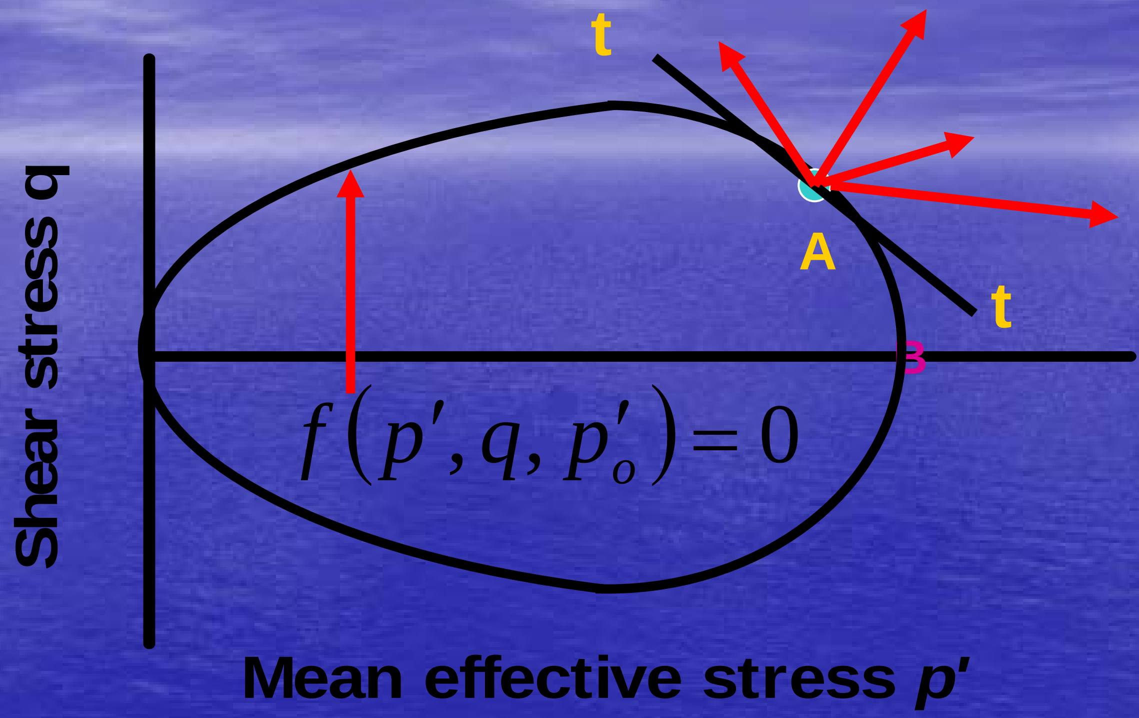
Yield surface:

$$f\left(p', q, p'_o\right) = 0$$



Size of the yield surface; usually $p'_o = p'$ for the isotropic stress state on the surface.

Flow rule or plastic potential



For a stress state on yield surface and with stress increment pointing outside the surface, plastic deformation occurs.

Flow rule

direction of plastic strain increment.

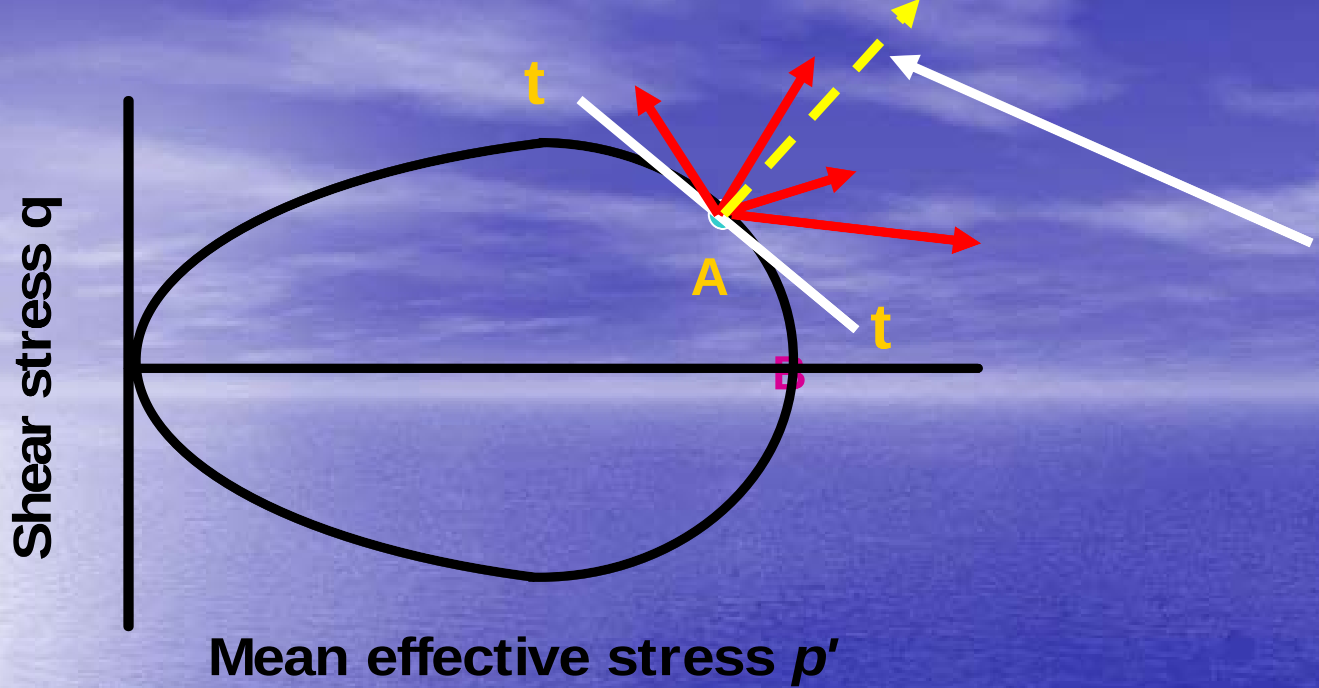
$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p}$$

Plastic volumetric strain increment

Plastic shear strain increment

Experiment & theory study in metal deform.
➔ direction of plastic strain increment dependent on the stress state, independent of stress increment.

The same for soil



$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p}$$

Flow rule

independent of stress increment

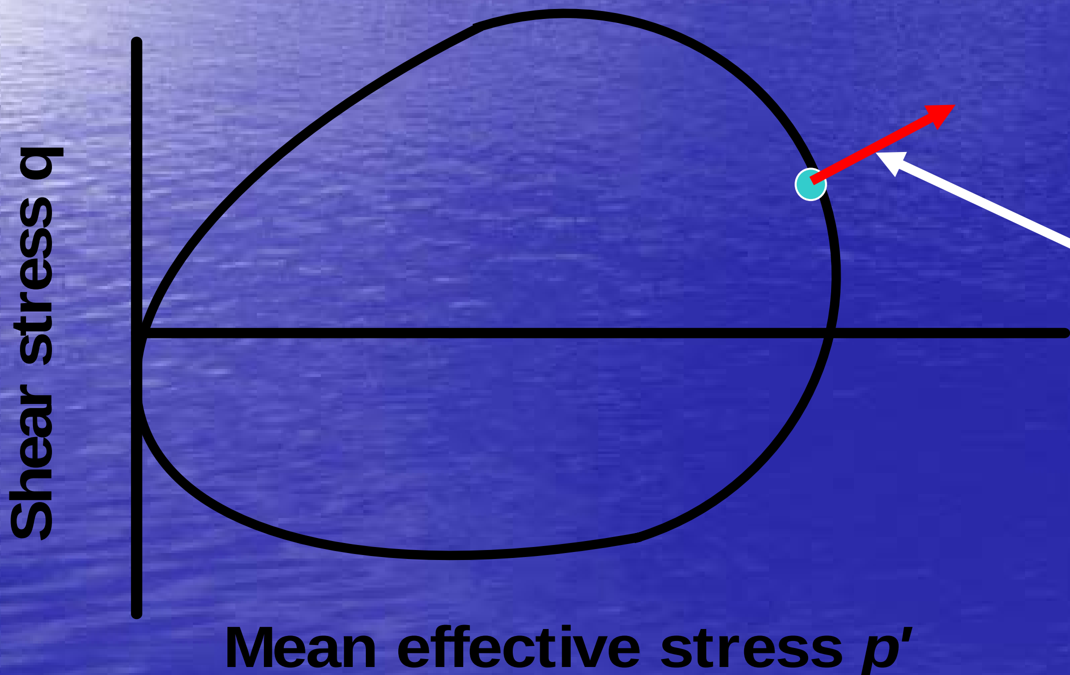
For a given stress sate

$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = r(p', q)$$

Plastic potential

Plastic potential $g(p',q)$ defined as:

$$\frac{\partial g / \partial p'}{\partial g / \partial q} = \frac{d\varepsilon_v^p}{d\varepsilon_d^p} = r(p', q)$$



$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = r(p'_B, q)$$

Yield surface & plastic potential for original Cam Clay model

Energy dissipation equation

Very importance concept, a fundamental law in natural science, a law in a higher order than constitutive relations.

Energy dissipation of soil proposed

Soil is a frictional material and the dissipation of the plastic energy is proportional to distortional strain increment and the mean effective stress acting on it.

Yield surface & plastic potential for original Cam Clay model

The dissipation of the plastic energy is proportional to distortional strain increment and the mean effective stress acting on it.

Plastic work input to an element of soil:
Stress parameter *p′* and *q* and strain $d\varepsilon_v^p$ and $d\varepsilon_d^p$ work- conjugated.

$$dW = p'd\varepsilon_v^p + qd\varepsilon_d^p$$

➔

$$p'd\varepsilon_v^p + qd\varepsilon_d^p = \mathrm{M}p'd\varepsilon_d^p$$

Yield surface & plastic potential for original Cam Clay model

The dissipation of the plastic energy is proportional to distortional strain increment and the mean effective stress acting on it.

$$p' d\varepsilon_v^p + q d\varepsilon_d^p = \mathrm{M} p' d\varepsilon_d^p$$

a thermodynamic material constant defining the rate of energy dissipation

Drucker's stability criterion (1952)

The work done by the external agency on the change in displacement it produces must be positive or zero

B

An element of soil considered here.
The element, an original equilibrium system, the stress (*p′*, *q*); and the displacement (dεv, dεd).

An additional force, external agency, (d*p′*, d*q*) applied , and the displacement it produces is ($d\varepsilon_v$, $d\varepsilon_d$).

$$\int \left(dp' d\varepsilon_v + dq d\varepsilon_d \right) \geq 0$$

Drucker's stability criterion (1952)

The work done by the external agency on the change in displacement it produces must be positive or zero

B

For an enclosed stress path, we obtain by integration

$$\oint \left(dp' d\varepsilon_v + dq d\varepsilon_d \right) \geq 0$$

The deformation made up of elastic part and plastic part

$$d\varepsilon = d\varepsilon^e + d\varepsilon^p$$

Elastic deformation completely recoverable, no energy dissipation

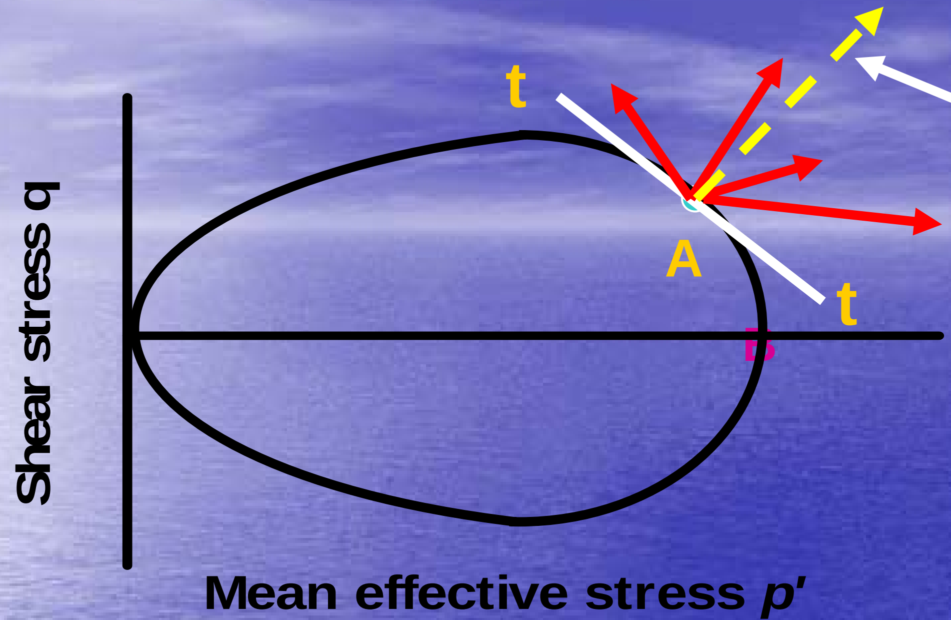
Drucker's stability criterion and normality

According to Drucker's stability criterion

B

$$\oint \left(dp' d\varepsilon_v^p + dq d\varepsilon_d^p \right) \geq 0$$

Drucker's stability criterion and normality



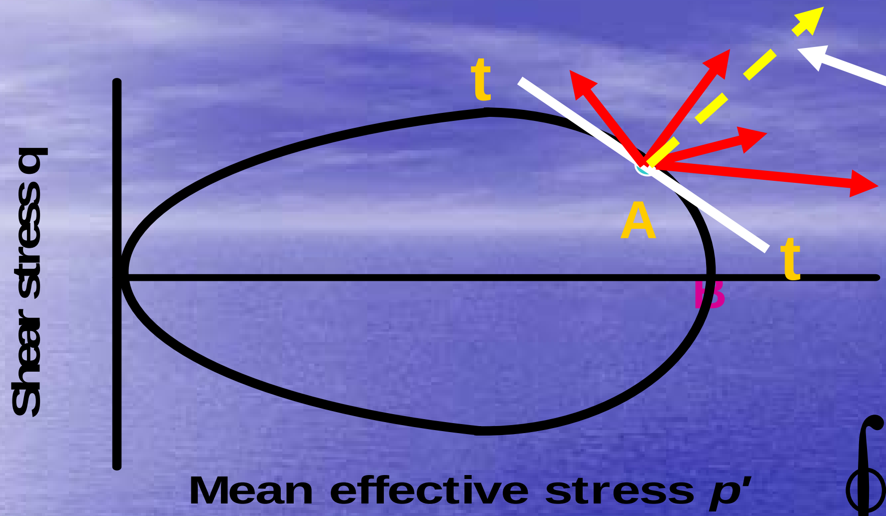
$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p}$$

B

$$\oint\left(dp'd\varepsilon_v^p + dqd\varepsilon_v^p\right) \geq 0$$

For all stress paths the stability criterion satisfied
tt: the tangential line
All the paths outside tt line cause plastic deformation

Drucker's stability criterion and normality



$$\frac{d\,\varepsilon_v^{\,p}}{d\,\varepsilon_d^{\,p}}$$

B

$$\oint\left(dp'd\varepsilon_v^p + dqd\varepsilon_v^p\right) \geq$$

➔ **Plastic strain increment vector MUST BE normal to the yield surface**

If not, negative work will be produced. The angle between stress increment along tt line and dε^p>90°.

Normality

Drucker's stability criterion and normality

Plastic strain increment normal to the yield surface

Associated flow rule: Normality

Non-associated flow rule: plastic strain increment NOT normal to the yield surface.

For associated flow rule:

$$g\left(p', q, p'_o\right) = f\left(p', q, p'_o\right)$$

plastic potential **yield surface**

For soil with associated flow rule

$$g\left(p', q, p'_o\right) = f\left(p', q, p'_o\right)$$

$$\Rightarrow \quad \frac{d\varepsilon_v^p}{d\varepsilon_d^p} = \frac{\partial f / \partial p'}{\partial f / \partial q}$$

B

With energy dissipation equation

$$p' d\varepsilon_v^p + q d\varepsilon_d^p = \mathrm{M} p' d\varepsilon_d^p$$

Yield surface & plastic potential

$$p' d\varepsilon_v^p + q d\varepsilon_d^p = \mathrm{M} p' d\varepsilon_d^p$$

By solving differential equations

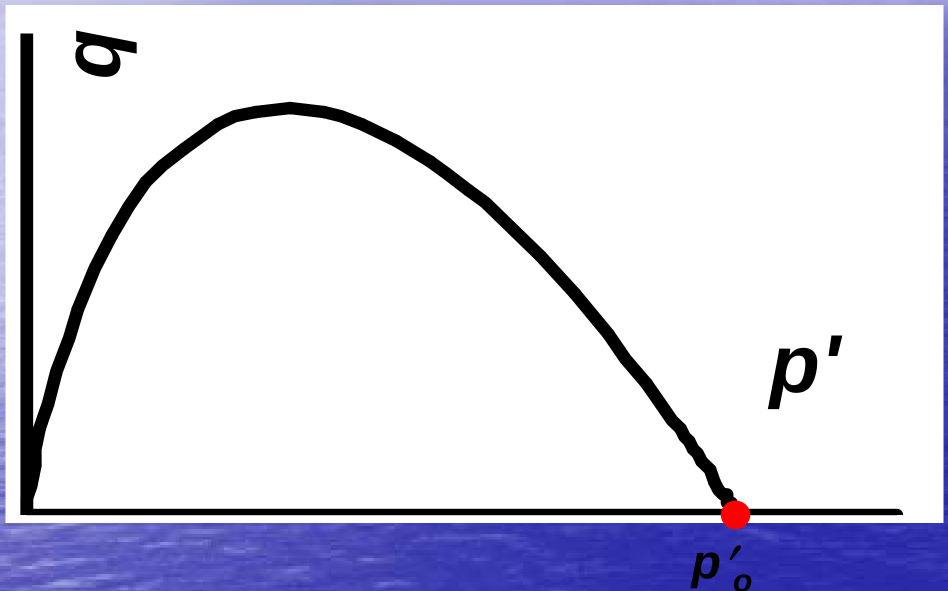
B

$$f\left(p', q, p_c'\right) = g\left(p', q, p_c'\right) = \frac{q}{\mathrm{M}p'} - \ln\left(\frac{p_o'}{p'}\right) = 0$$

defining the rate of energy dissipation: a fundamental material property.

Yield surface

$$f\left(p', q, p'_c\right) = \frac{q}{Mp'} - \ln\left(\frac{p'_o}{p'}\right) = 0$$



p'_o: size of the yield surface

For isotropic loading, $\sigma'_1 = \sigma'_2 = \sigma'_3$,

then $p' = p'_o$.

Flow rule

$$\eta = \frac{q}{p'}$$

Shear stress ratio

B

$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = \frac{\partial g / \partial p'}{\partial g / \partial q} = \mathrm{M} - \eta$$

➔ $$d\varepsilon_d^p = \frac{d\varepsilon_v^p}{\left(\dfrac{\partial g / \partial p'}{\partial g / \partial q}\right)} = \frac{d\varepsilon_v^p}{(\mathrm{M} - \eta)}$$

Hardening of soil

$$d\varepsilon_d^p = \frac{d\varepsilon_v^p}{\left(\frac{\partial g / \partial p'}{\partial g / \partial q}\right)} = \frac{d\varepsilon_v^p}{(\mathrm{M} - \eta)}$$

B

If plastic volumetric deformation $d\varepsilon_v^p$ known, plastic shear deformation $d\varepsilon_d^p$ determined

Hardening of soil: enlargement of yield surface with plastic deformation

Seek a relationship between the enlargement of yield surface and the plastic deformation

Hardening of the yield surface

A fundamental assumption for derivation

The hardening of yield surface is dependent on the plastic volumetric deformation only.

Consequently,

(1) Plastic volumetric deformation determined by change in size of yield surface irrespective of loading stress paths.

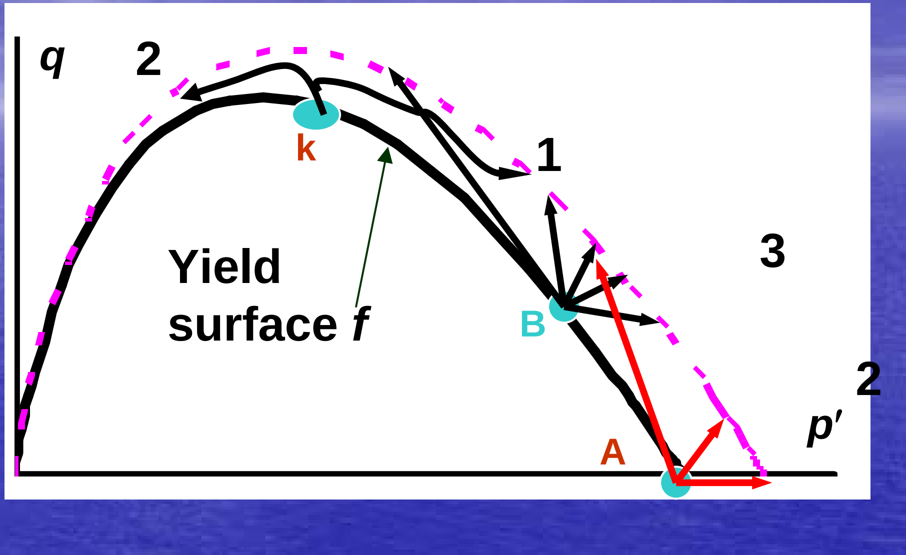
➔

$$d\varepsilon_v^p = \Lambda dp_o'$$

(2) one to one relationship between size of yield surface and plastic volumetric deformation.

Hardening of the yield surface

$$d\varepsilon_v^p = \Lambda \, dp'_o$$



at stress A:
Three stress increment
➔the same dp'_o
➔the same dε_v^p

All the loading, on yield surface *f*, resulting the same change in the yield surface *f*➔ the same plastic volumetric strain

Soil deformation during isotropic loading well studied.
We try to work out relationship between dp'_o and dε_v^p obtained isotropic compression behaviour of soils.

Hardening of the yield surface

For isotropic compression

Stress state in the p'-q space: op' axis.

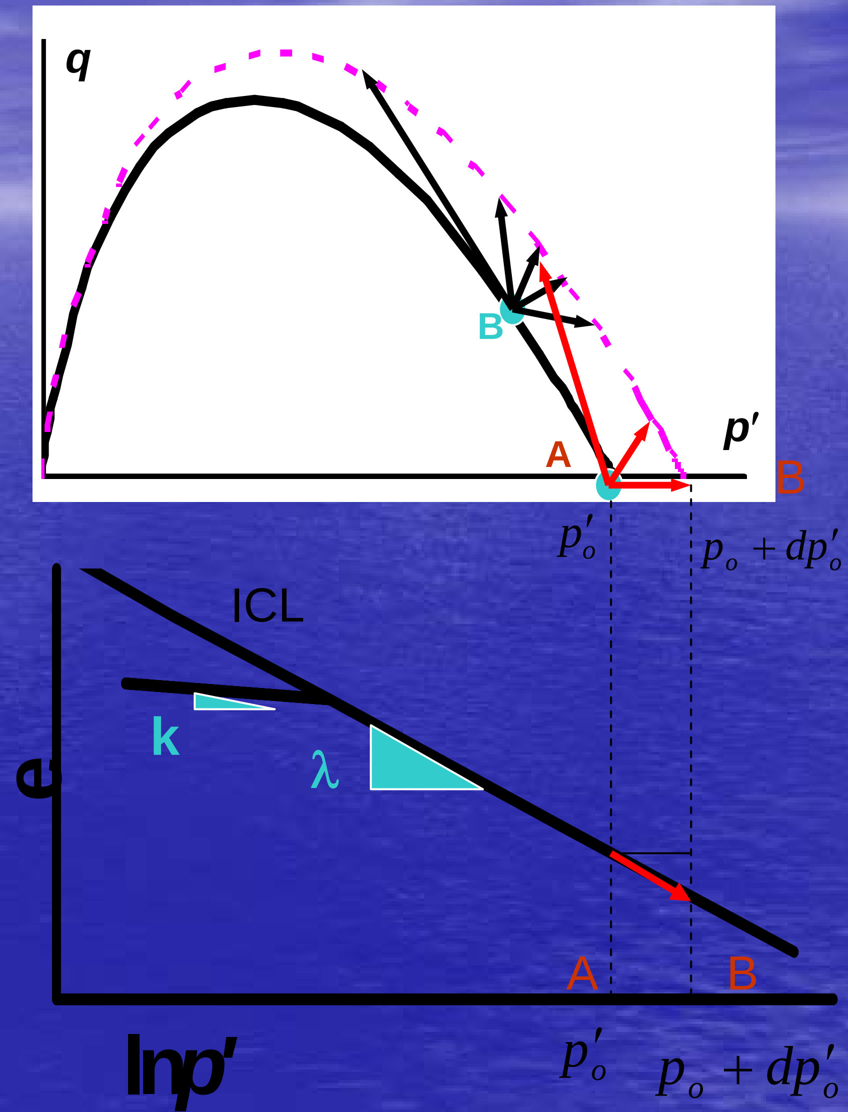
Soil state in the e-lnp' space ICL.

At stress state A ($p'=p'_a$, $q=0$)

With an stress increment dp' to B (p'_a + dp').

For voids ratio at A: e_A,

At B: e_A + de



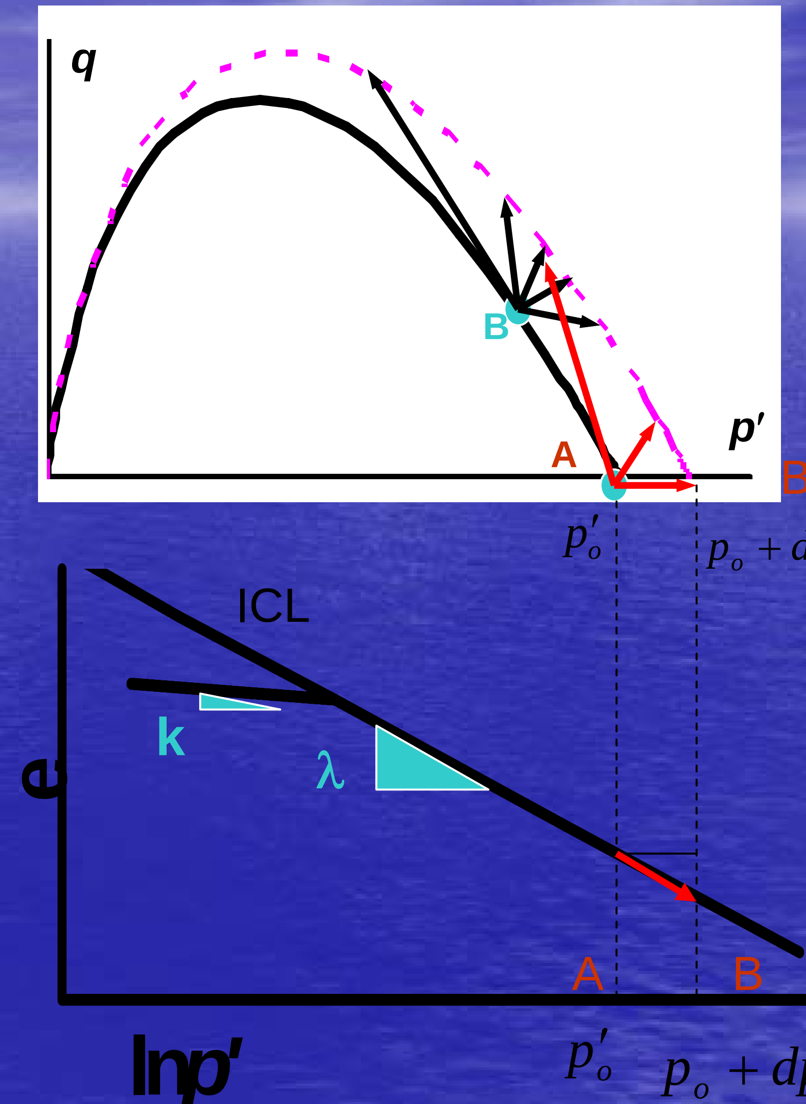
Hardening of the yield surface

For Isotropic Compression Line,
Virgin compression: λ
Elastic compression: κ

For compression from A and B: Virgin compression

The reduction in voids ratio

$$de = -\lambda \Delta(\ln p') = -\frac{\lambda}{p'} dp'$$



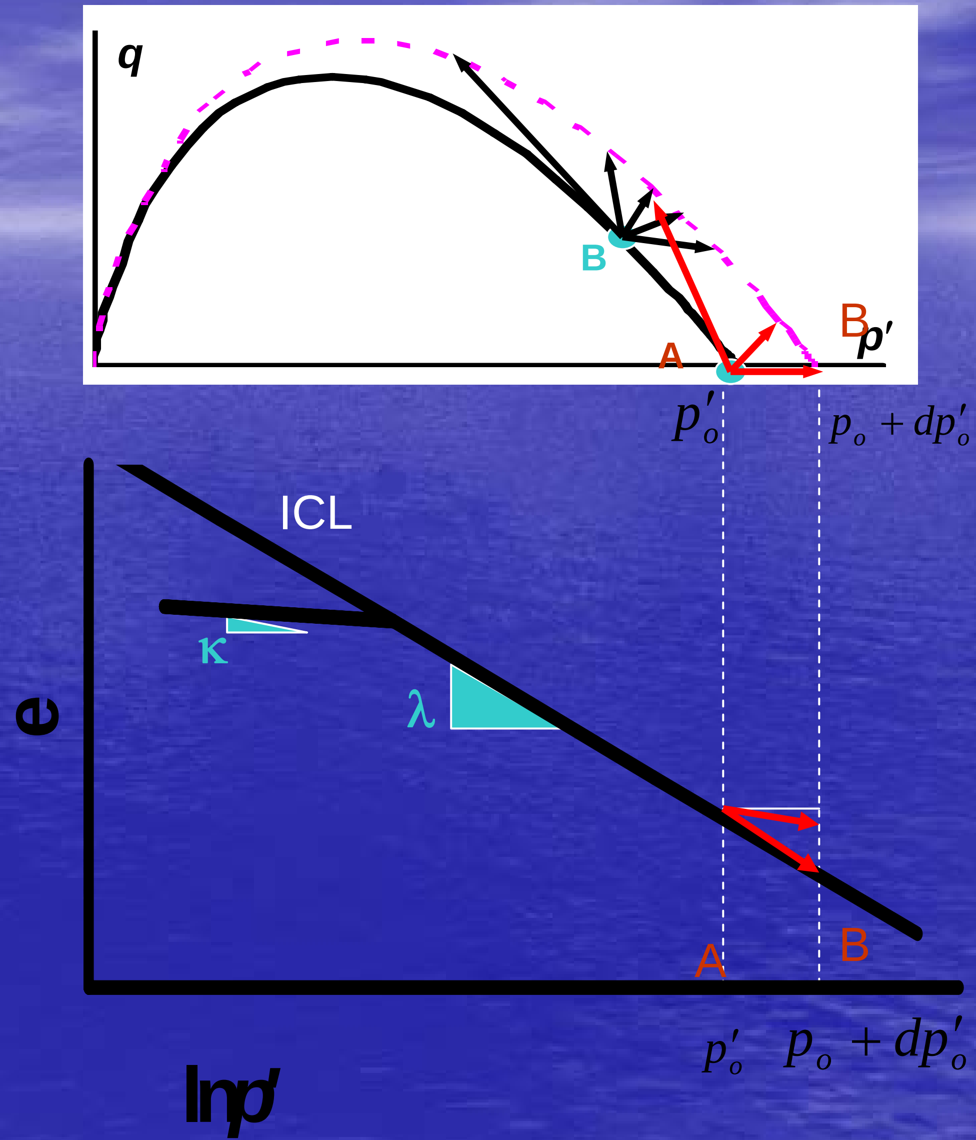
Soil compression along ICL

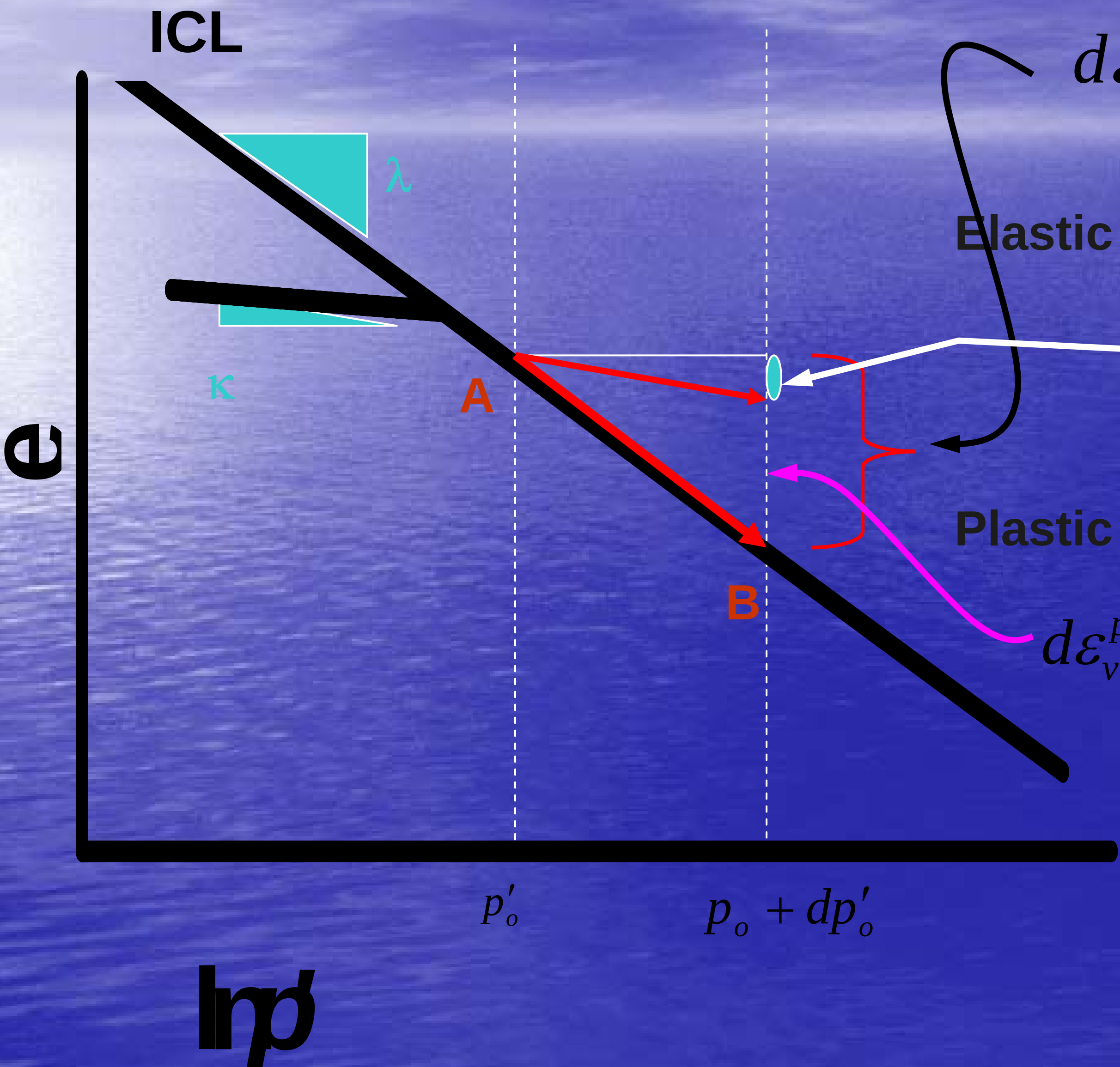
The total volumetric strain increment

$$d\varepsilon_v = -\frac{de}{1+e} = \frac{\lambda}{(1+e)p'}dp'$$

Elastic volumetric strain increment
By Hook's law
➔

$$d\varepsilon_v^e = \frac{\kappa}{(1+e)p'}dp'$$





The total volumetric strain increment

$$d\varepsilon_v = \frac{\lambda}{(1+e)p'}dp'$$

Elastic volumetric strain increment

$$d\varepsilon_v^e = \frac{\kappa}{(1+e)p'}dp'$$

Plastic volumetric strain increment

$$d\varepsilon_v^p = d\varepsilon_v - d\varepsilon_v^e = \frac{(\lambda-\kappa)}{(1+e)p'}dp'$$

Hardening of the yield surface

Soil compresses along ICL,

The total volumetric strain increment

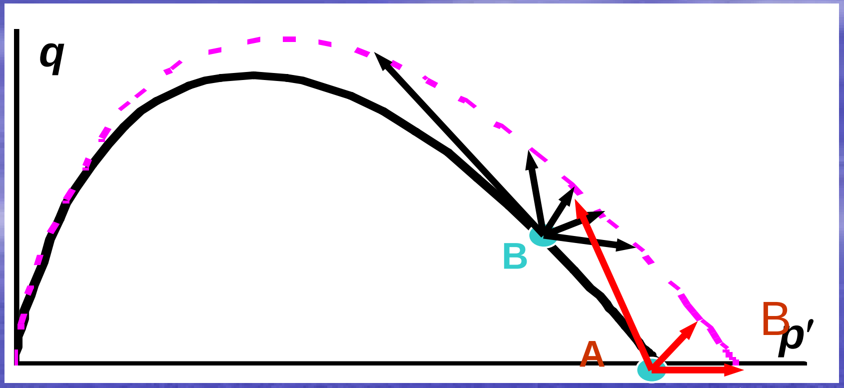
$$d\varepsilon_v = \frac{\lambda}{(1+e)p'}dp'$$

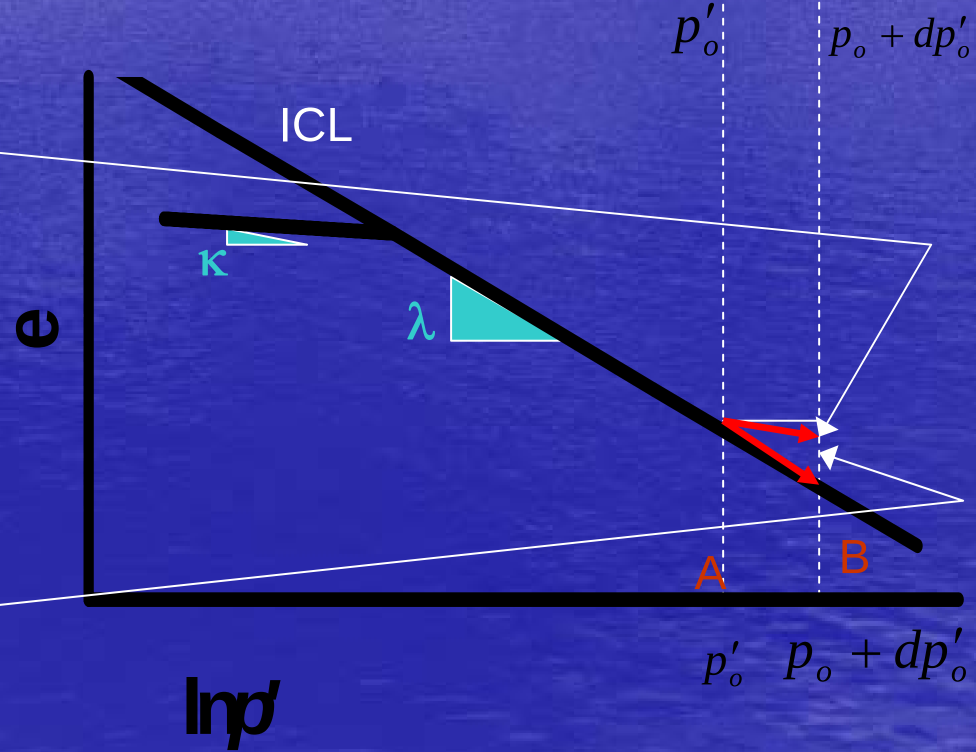
Elastic volumetric strain increment

$$d\varepsilon_v^e = \frac{\kappa}{(1+e)p'}dp'$$

Plastic volumetric strain increment

$$d\varepsilon_v^p = d\varepsilon_v - d\varepsilon_v^e = \frac{(\lambda-\kappa)}{(1+e)p'}dp'$$





Hardening of the yield surface

$d\varepsilon_v^p$ obtained for yielding during isotropic compression

$$d\varepsilon_v^p = \frac{(\lambda - \kappa)}{(1+e)p'} dp'$$

This equation is derived for ICL and valid for ICL only. Plastic volumetric deformation expressed in term of the mean effective stress and the increment in mean effective stress. Because for IC, $q \equiv 0$.

Hardening of the yield surface

To obtain plastic volumetric deformation equation for loading along general stress path

The hardening of yield surface is dependent on the plastic volumetric deformation only.

Equation for $d\varepsilon_v^p$ should be written in terms of size of yield surface p'_o, and change of yield surface in size dp'_o.

Hardening of the yield surface

We know, for Isotropic compression,

p′o= p′ and dp′o= dp′

➔ **From** $$d\varepsilon_v^p = \frac{(\lambda - \kappa)}{(1+e)p'}dp'$$ **to** $$d\varepsilon_v^p = \frac{(\lambda - \kappa)}{(1+e)p'_o}dp'_o$$

The equation valid for all stress increments that result in the same change in the yield surface

Plastic deformation

Yield surface

$$f\left(p', q, p'_c\right) = \frac{q}{\mathrm{M}p'} - \ln\left(\frac{p'_o}{p'}\right) = 0$$

Flow rule

$$f\left(p', q, p'_c\right) = g\left(p', q, p'_c\right) \qquad \frac{d\varepsilon_v^p}{d\varepsilon_d^p} = \left(\mathrm{M} - \eta\right)$$

Hardening of the yield surface

$$d\varepsilon_v^p = \frac{\left(\lambda - \kappa\right)}{\left(1+e\right)p'} dp'_o$$

Plastic deformation completely defined

From the yield surface function

$$dp'_o = p'_o \left\{ \left[1 - \ln\left(\frac{p'_o}{p'}\right) \right] \frac{dp'}{p'} + \frac{dq}{\mathrm{M}p'} \right\}$$

Plastic deformation

Yield surface

$$f\left(p', q, p'_c\right) = \frac{q}{\mathrm{M}p'} - \ln\left(\frac{p'_o}{p'}\right) = 0$$

Flow rule

$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = \left(\mathrm{M} - \eta\right)$$

Hardening of the yield surface

$$d\varepsilon_v^p = \frac{\left(\lambda - \kappa\right)}{\left(1+e\right)p_o} dp'_o$$

Plastic deformation obtained

$$\begin{cases} d\varepsilon_v^p = \dfrac{\left(\lambda-\kappa\right)}{\left(1+e\right)}\left\{\left[1-\ln\left(\dfrac{p'_o}{p'}\right)\right]\dfrac{dp'}{p'} + \dfrac{dq}{\mathrm{M}p'}\right\} \\ d\varepsilon_d^p = \dfrac{\left(\lambda-\kappa\right)}{\left(1+e\right)\left(\mathrm{M}-\eta\right)}\left\{\left[1-\ln\left(\dfrac{p'_o}{p'}\right)\right]\dfrac{dp'}{p'} + \dfrac{dq}{\mathrm{M}p'}\right\} \end{cases}$$

Total increment stress and strain relationship

Plus elastic deformation, total increment stress and strain relationship

$$\begin{cases} d\varepsilon_v = \dfrac{\kappa}{(1+e)}\dfrac{dp'}{p'} + \dfrac{(\lambda-\kappa)}{(1+e)}\left\{\left[1-\ln\left(\dfrac{p'_o}{p'}\right)\right]\dfrac{dp'}{p'} + \dfrac{dq}{\mathrm{M}p'}\right\} \\ d\varepsilon_d = \dfrac{2\kappa(1+\nu)}{9(1+e)(1-2\nu)}\dfrac{dq}{p'} + \dfrac{(\lambda-\kappa)}{(1+e)(\mathrm{M}-\eta)}\left\{\left[1-\ln\left(\dfrac{p'_o}{p'}\right)\right]\dfrac{dp'}{p'} + \dfrac{dq}{\mathrm{M}p'}\right\} \end{cases}$$

Elastic **plastic**

Now that **The change in the strain state of an element of material to the change in the stress state acting on the element.**

defined. A model is completed.

The model is the original Cam Clay model.

Original Cam Clay model

Unify consistently the mechanical properties of soil into a simple and elegant theoretical framework.
Most important development in modern soil mechanics

Two distinguished features of the original Cam Clay model and models of the Cam Clay family

(1) Volumetric dependent of hardening

The hardening of yield surface is dependent on the plastic volumetric deformation only.

Plastic volumetric deformation uniquely determined by the change of yield surface in size; and
There is one to one relationship between the size of the yield surface and plastic volumetric deformation.

(2) Prediction of the existence of a critical state of deformation

Let's examine flow rule and energy dissipation function

$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = (\mathrm{M} - \eta)$$

$$p' d\varepsilon_v^p + q d\varepsilon_d^p = \mathrm{M} p' d\varepsilon_d^p$$

At a special stress state, $\eta = \mathrm{M}$, from flow rule,

$$d\varepsilon_d^p = \frac{d\varepsilon_v^p}{(\mathrm{M} - \eta)} = \infty$$

At this special state, soil has no resistance to shear deformation. Like water or gas, it can not resist distortion.

(2) Prediction of the existence of a critical state of deformation

At this special state, η=M, from the energy dissipation.

$$p'd\varepsilon_v^p + qd\varepsilon_d^p = \mathrm{M}p'd\varepsilon_d^p$$

For no change of $d\varepsilon_v^p$, i.e., $d\varepsilon_v^p = 0$, then

$$qd\varepsilon_d^p = \mathrm{M}p'd\varepsilon_d^p$$

any value of $d\varepsilon_d^p$ can satisfy the equation.

➔Soil can remain at the state η=M with no change in its stress, $dp'_o = 0$, and no plastic volumetric deformation. However, the shear plastic deformation can be infinitive.

This is a Critical State of deformation

Original Cam Clay model

A Critical State of deformation:

At a Critical State of deformation, a soil has no resistance to shear deformation and the soil can be distorted continuously with its stress state and voids ratio remain unchanged.

It is a final failure state.

Cam Clay model is the first Critical State model.
The theoretical framework, unifying consistently the mechanical properties of soil into a simple and elegantly by original Cam Clay model, the Critical State Soil Mechanics (CSSM).

CSSM established soil mechanics as a cohesive science.

The crown pearl of soil mechanics

5: Soil plasticity

From the introduction of the original Cam Clay model basics of soil plasticity are introduced

(1) Soil deformation divided into elastic and plastic parts

$$d\varepsilon = d\varepsilon^{e} + d\varepsilon^{p}$$

(2) Elastic deformation: Hook's law

$$d\varepsilon_1 = \frac{d\sigma_1'}{E_1} - \nu_{21}\frac{d\sigma_2'}{E_2} - \nu_{31}\frac{d\sigma_3'}{E_3}$$

Deformation in direction 1

5: Soil plasticity

(3) Modelling plastic

Three componants required:

(i) Yield surface, (ii) Flow rule and (iii) Hardening function

(i) Yield surface in the stress space

Size of the yield surface

$$f\left(p', q, p'_o\right) = 0$$

dividing soil behaviour into two regions: loading inside the surface: elastic, loading on the surface, plastic.

During plastic deformation, the yield surface changes with plastic deformation and the current stress state remains on the yield surface.

5: Soil plasticity

Modelling plastic deformation

(ii) Flow rule: the direction of plastic strain increment

Direction of plastic strain increment dependent on the stress only, i.e., independent of the stress increment.

$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = r(p', q)$$

Direction of plastic strain increment normal to the yield surface: associated flow rule.

Direction of plastic strain increment NOT normal to yield surface: non-associated flow rule.

5: Soil plasticity

Modelling plastic deformation

(iii) Hardening of the yield surface

Variation of yield surface with plastic deformation.

For models of the Cam Clay family, hardening of yield surface: plastic volumetric deformation dependent

$$d\varepsilon_v^p = h\left(p', q\right) dp'_o$$

A more general hardening function can be dependent on the plastic strain as

$$dp'_o = h^{-1}\left(p', q, d\varepsilon_v^p, d\varepsilon_d^p\right)$$

5: Soil plasticity

Modelling plastic deformation

(i) Yield surface: $f\left(p', q, p'_o\right) = 0$

(ii) Flow rule: $\dfrac{d\varepsilon_v^p}{d\varepsilon_d^p} = r\left(p', q\right)$

(iii) Hardening of the yield surface $d\varepsilon_v^p = h\left(p', q\right) dp'_o$

With these three parts defined, plastic deformation found. From (i), the size change of the yield surface obtained as

$$dp'_o = -\frac{\dfrac{\partial f}{\partial p'} dp' + \dfrac{\partial f}{\partial q} dq}{\dfrac{\partial f}{\partial p'_o}}$$

5: Soil plasticity

Modelling plastic deformation

(i) Yield surface; (ii) Flow rule; (iii) Hardening of the yield surface

Then plastic strain increment can be expressed as

$$
\begin{cases}
d\varepsilon_v^p = -h(p',q) \times \dfrac{\dfrac{\partial f}{\partial p'}dp' + \dfrac{\partial f}{\partial q}dq}{\dfrac{\partial f}{\partial p'_o}} \\
d\varepsilon_v^p = -h(p',q)r(p',q) \times \dfrac{\dfrac{\partial f}{\partial p'}dp' + \dfrac{\partial f}{\partial q}dq}{\dfrac{\partial f}{\partial p'_o}}
\end{cases}
$$

Thank you very much!

6: Models of the Cam Clay family

Modified Cam Clay model

Dafalias model

Volumetric dependent hardening